Completely Reachable Automata

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(joint with Evgenija Bondar)

Ural Federal University, Ekaterinburg, Russia

DCFS'16, July 6, 2016

Where I am from?

Definitions, Terminology, and Disclaimer

We consider complete deterministic finite automata (DFAs).

$\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ where Q stands for the state set, Σ is the input alphabet, and $\delta : Q \times \Sigma \to Q$ is a (total) transition function.

To simplify notation we often introduce a DFA as $\langle Q, \Sigma \rangle$ and write $q \,.\, w$ for $\delta(q, w)$ and $P \,.\, w$ for $\{\delta(q, w) \mid q \in P\}$.

Given a DFA $\mathscr{A} = \langle Q, \Sigma \rangle$, a non-empty subset $P \subseteq Q$ is reachable in $\mathscr{A}$ if $P = Q \,.\, w$ for some word $w \in \Sigma^*$. A DFA is completely reachable if every non-empty set of its states is reachable.

The present talk is in fact a work-in-progress report—we do have some results but many, many natural questions still remain open. Some of these open questions will be mentioned here but many more can be found in the proceedings paper or in its expanded version at arXiv.org (arXiv:1607.00554).

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First Motivation: Synchronizing Automata

A $\mathscr{A} = \langle Q, \Sigma \rangle$ is called synchronizing if there is a word $w \in \Sigma^*$ whose action resets $\mathscr{A}$, that is, leaves $\mathscr{A}$ in one particular state no matter at which state it started: $q \,.\, w = q' \,.\, w$ for all $q, q' \in Q$.

In short, $|Q \,.\, w| = 1$; that is a singleton is reachable in $\mathscr{A}$.

Hence, a completely reachable automaton is synchronizing.

Any w with $|Q \,.\, w| = 1$ is a reset word for $\mathscr{A}$. The minimum length of reset words for $\mathscr{A}$ is called the reset threshold of $\mathscr{A}$.

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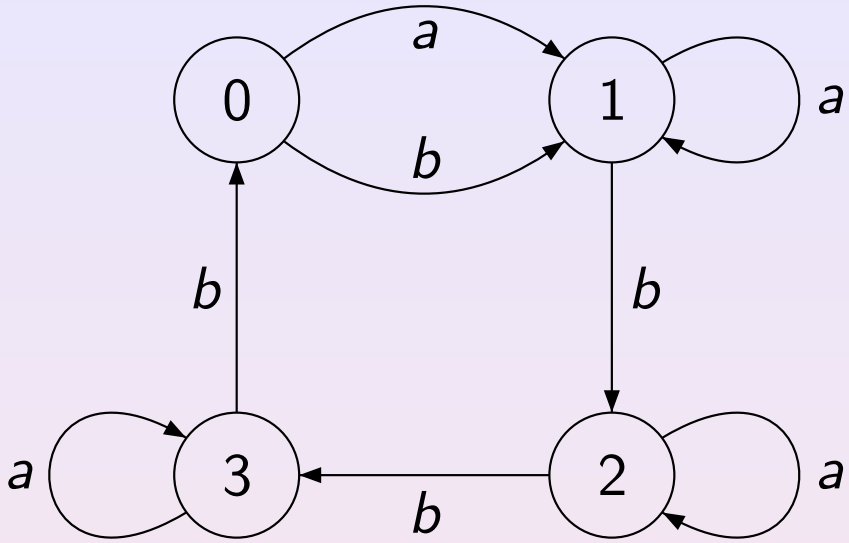

A reset word is *abbbabbba*: applying it at any state brings the automaton to the state 1.

In fact, this is the reset word of minimum length for the automaton whence its reset threshold is 9.

The automaton belongs to the series $\mathscr{C}_n$ found by Jan Černý in 1964. For each $n > 1$, the automaton $\mathscr{C}_n$ has n states, 2 input letters and reset threshold $(n-1)^2$.

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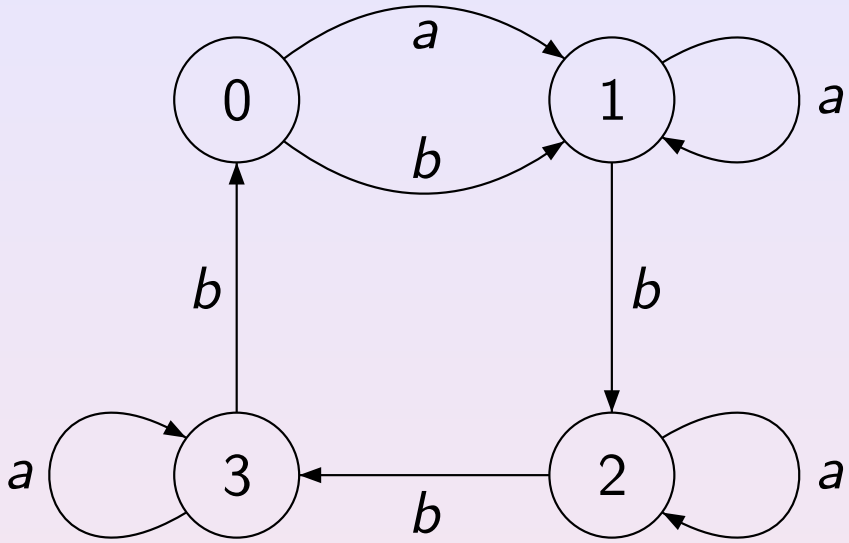

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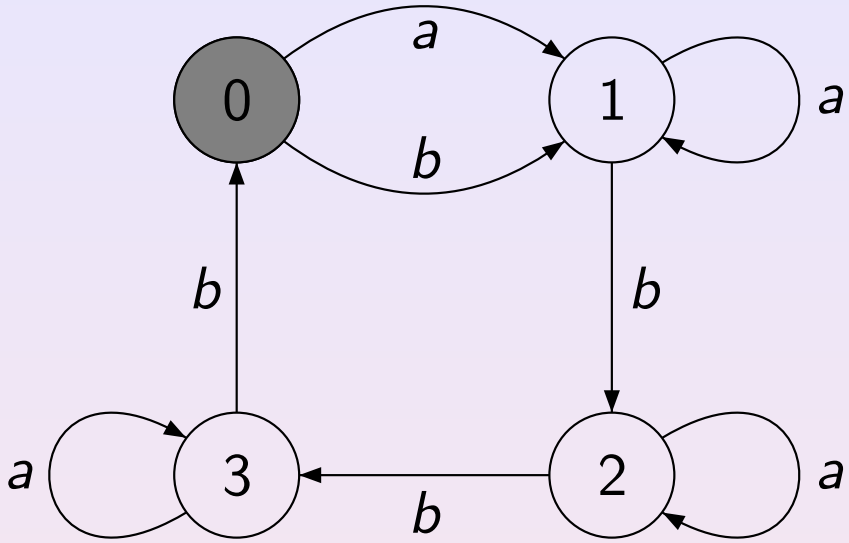

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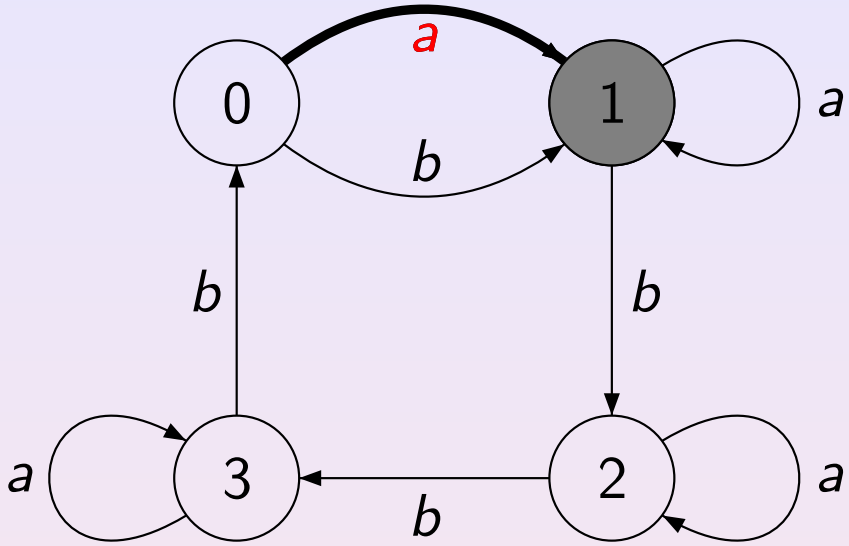

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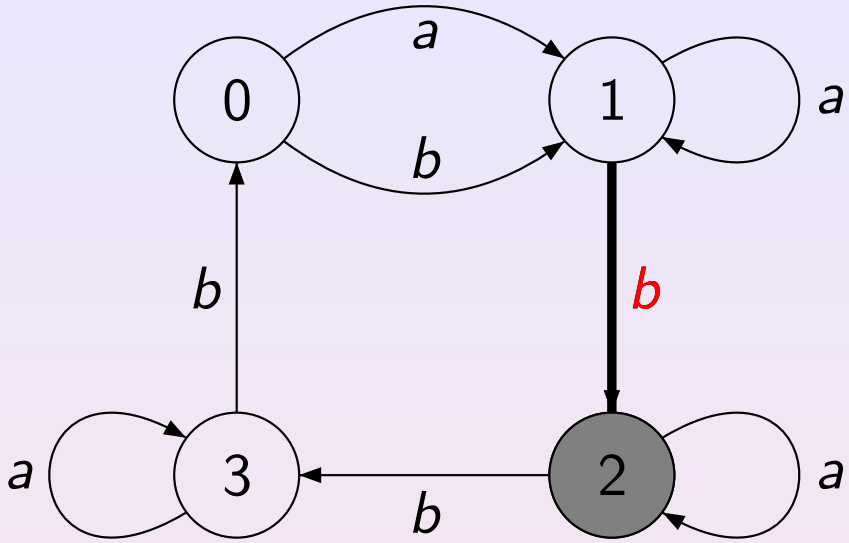

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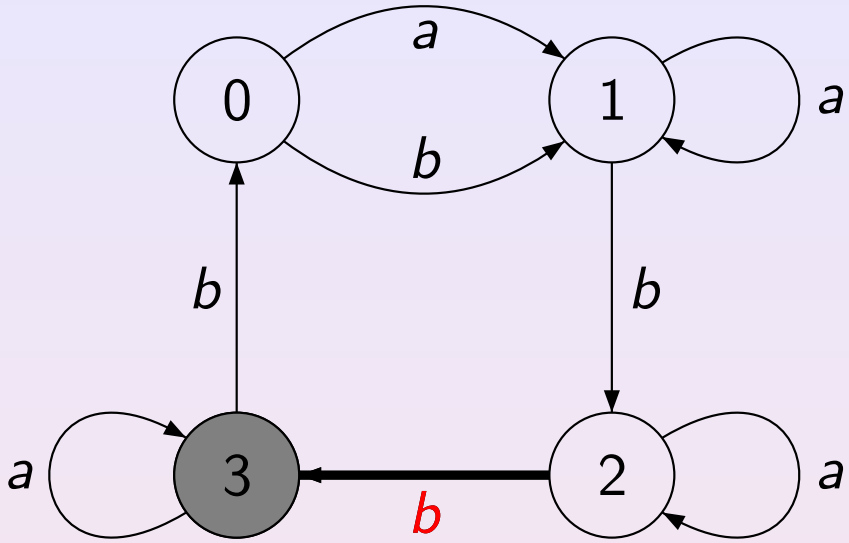

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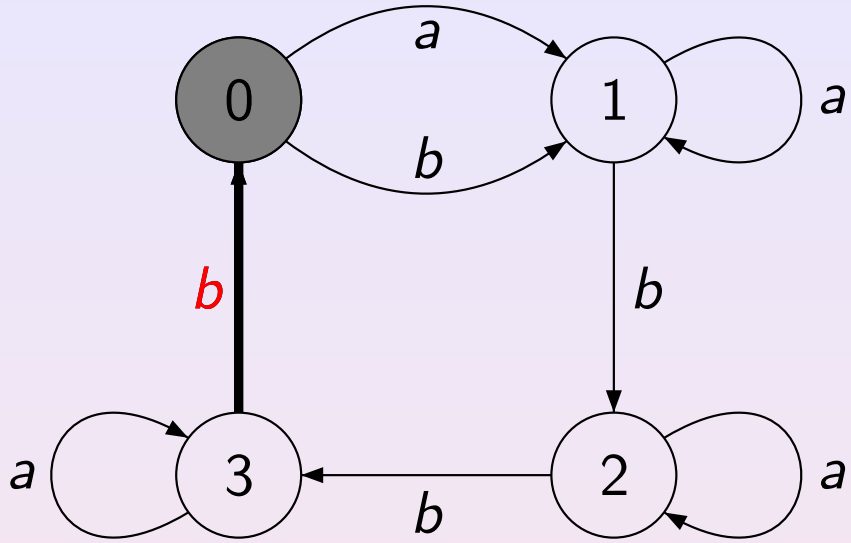

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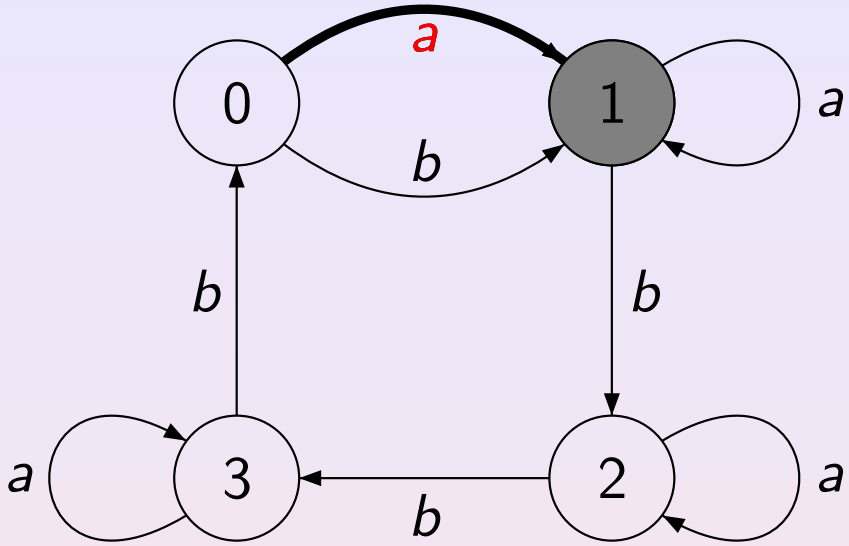

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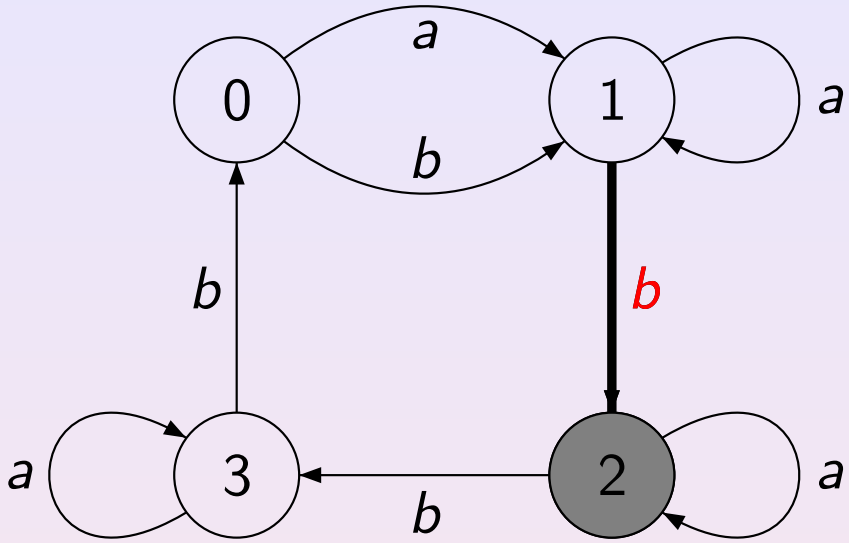

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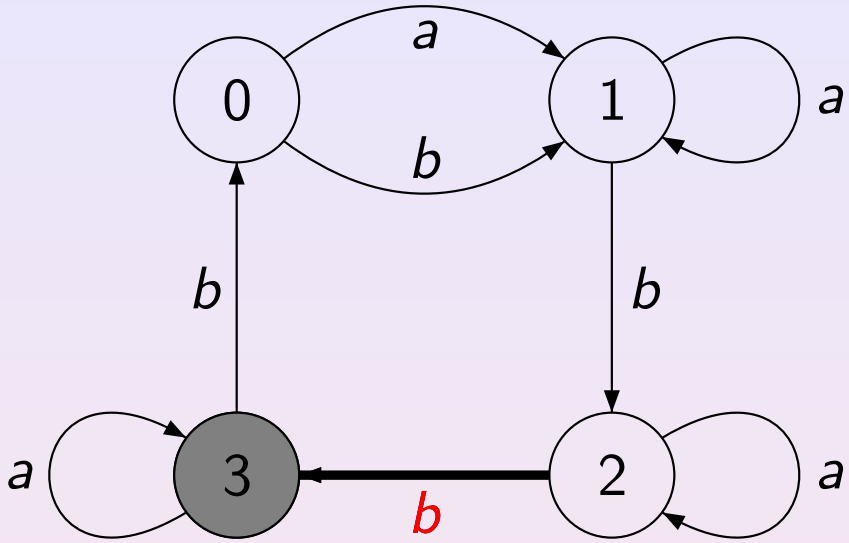

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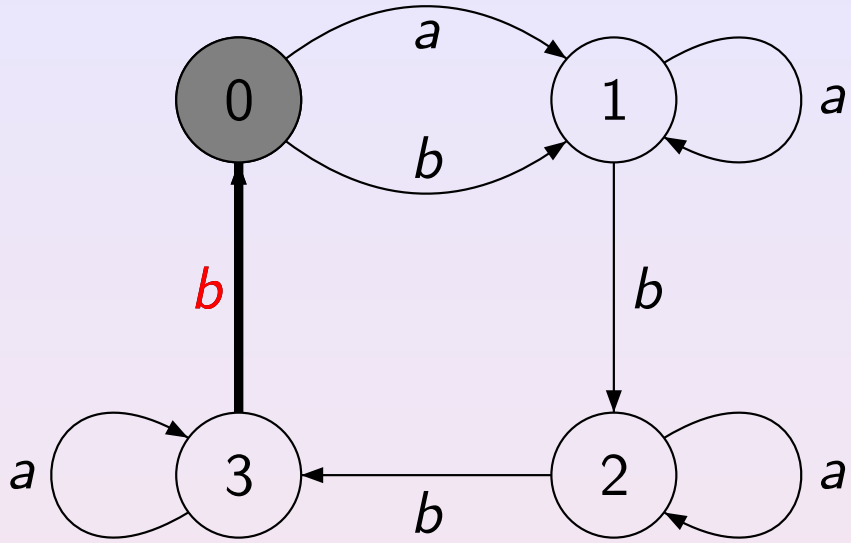

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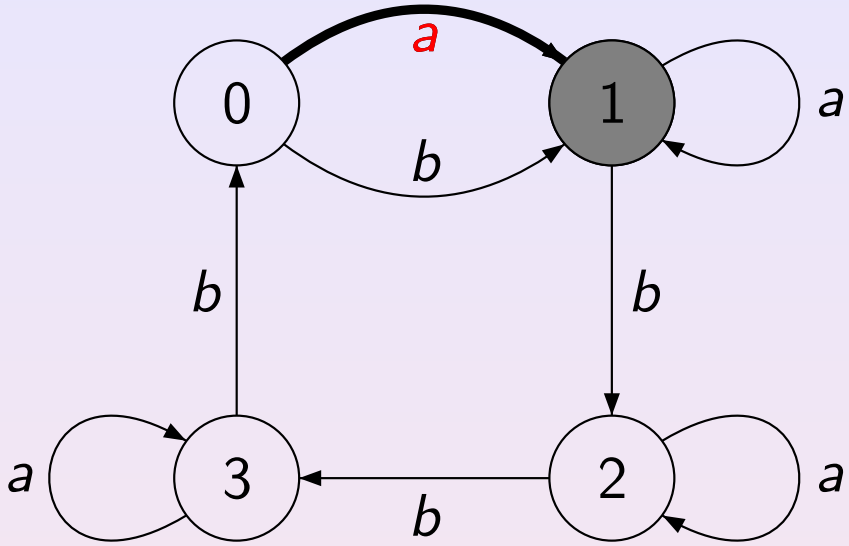

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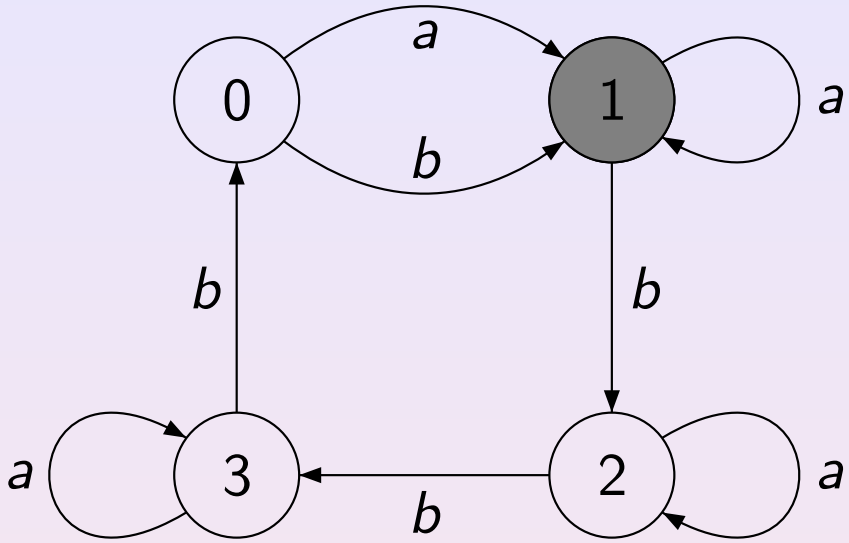

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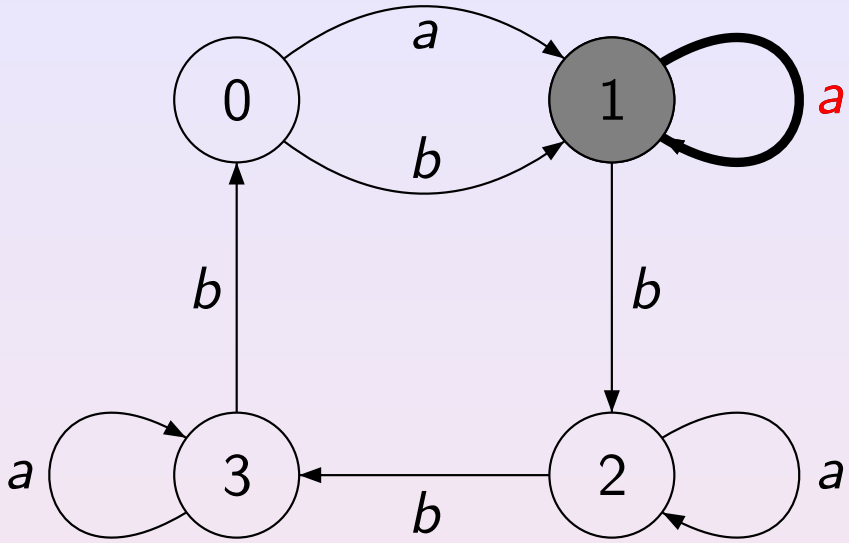

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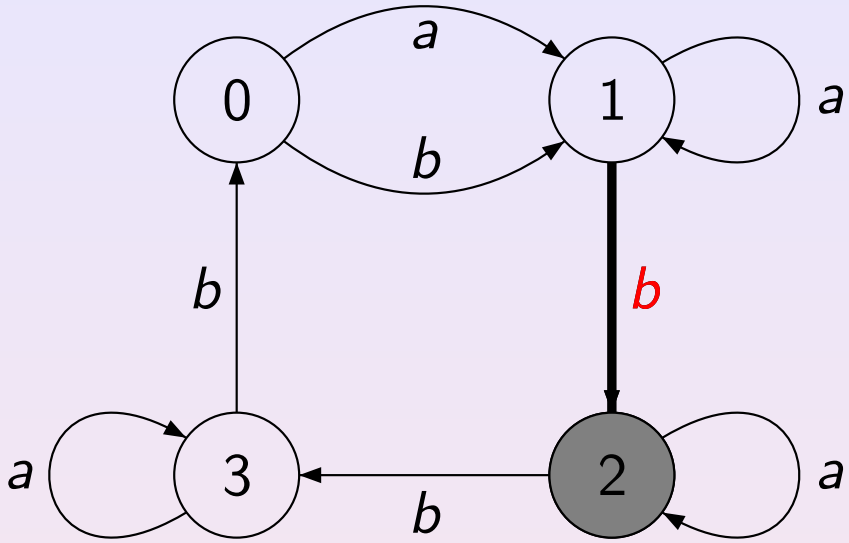

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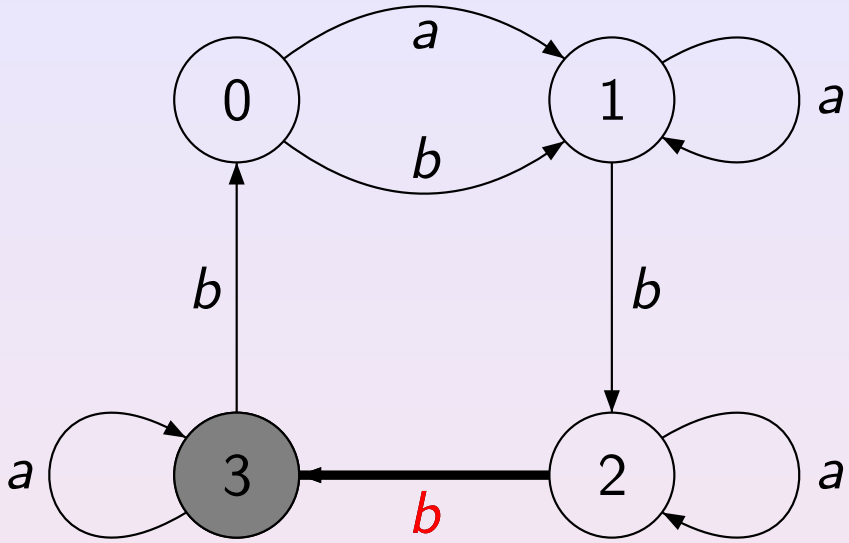

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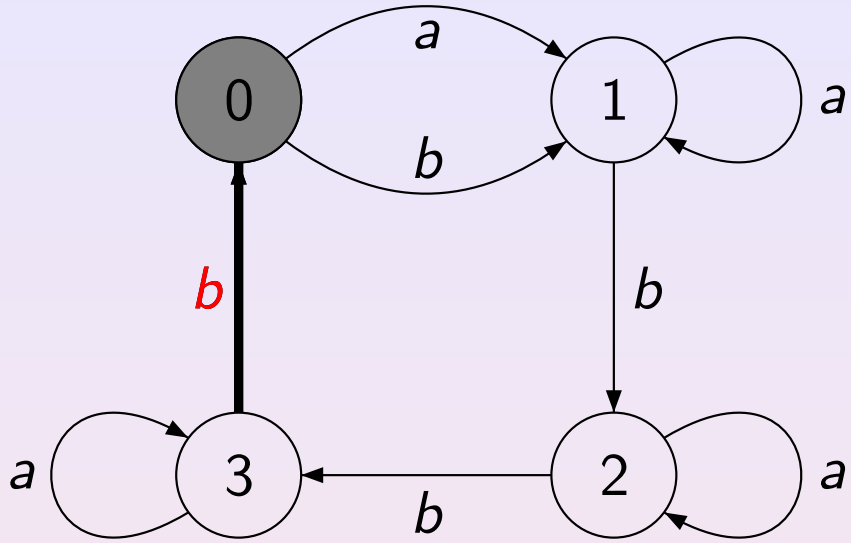

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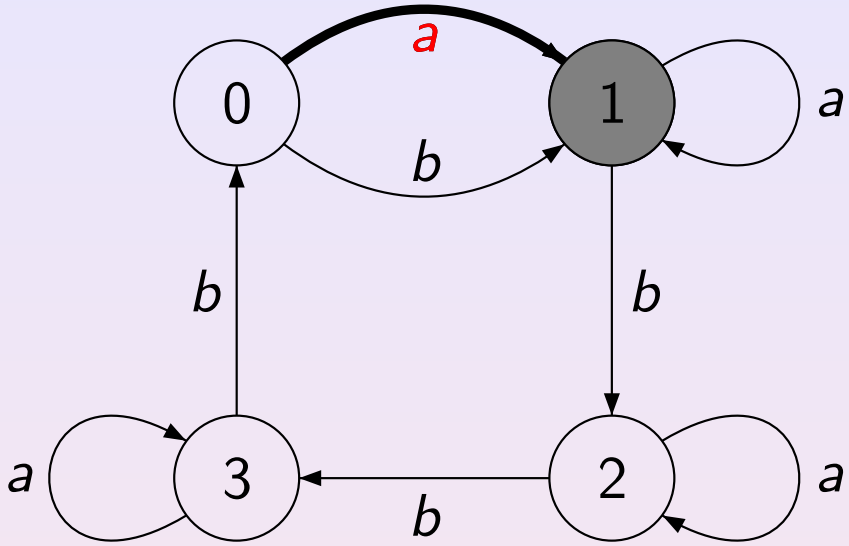

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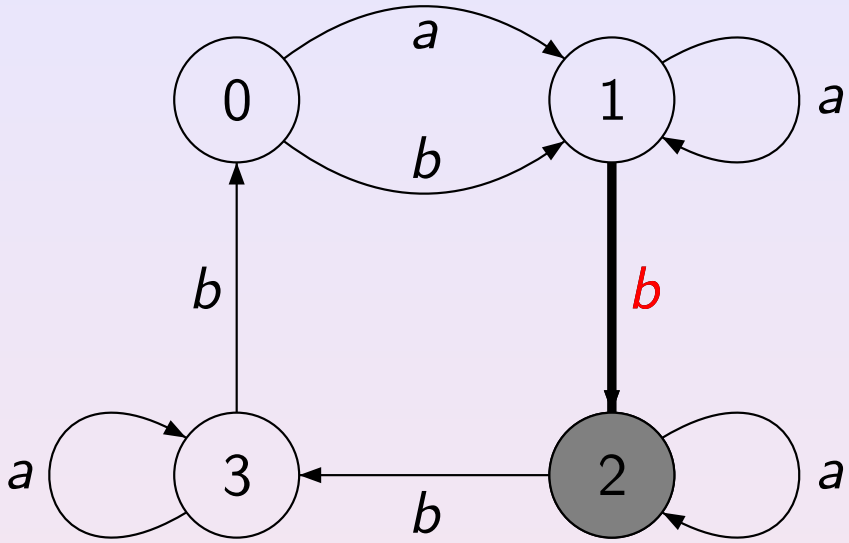

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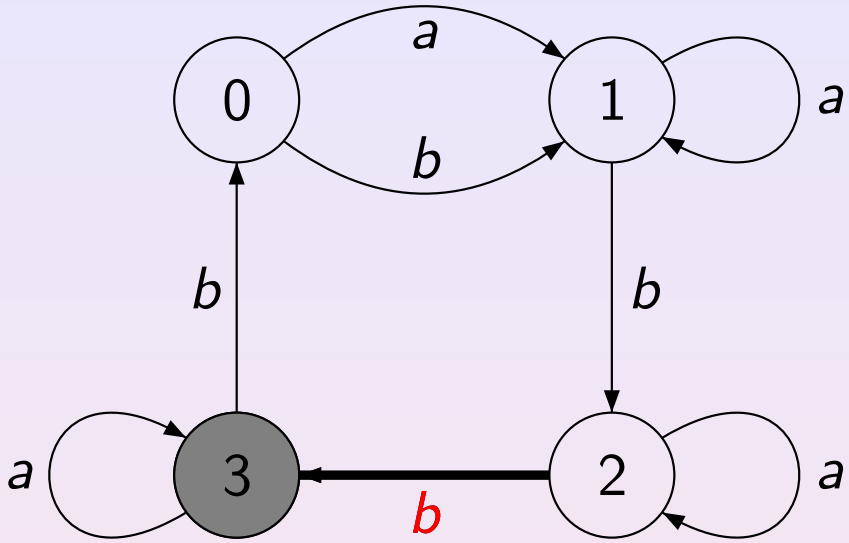

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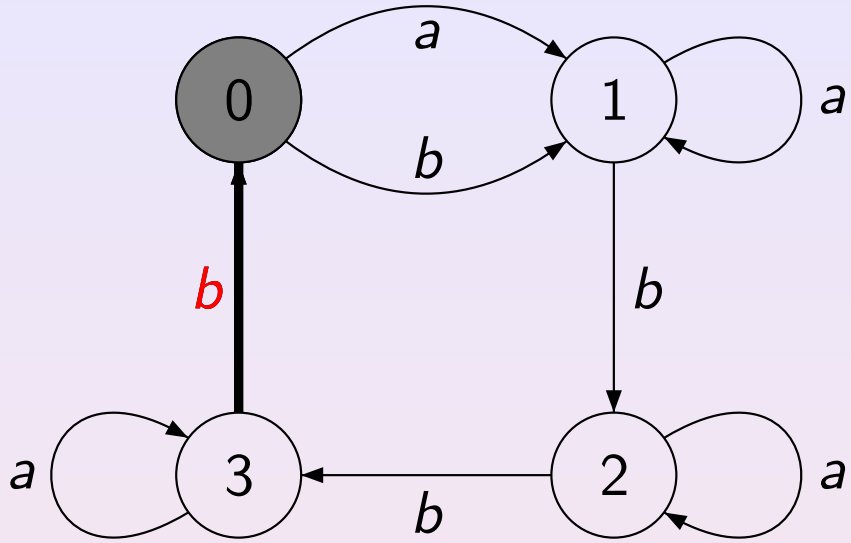

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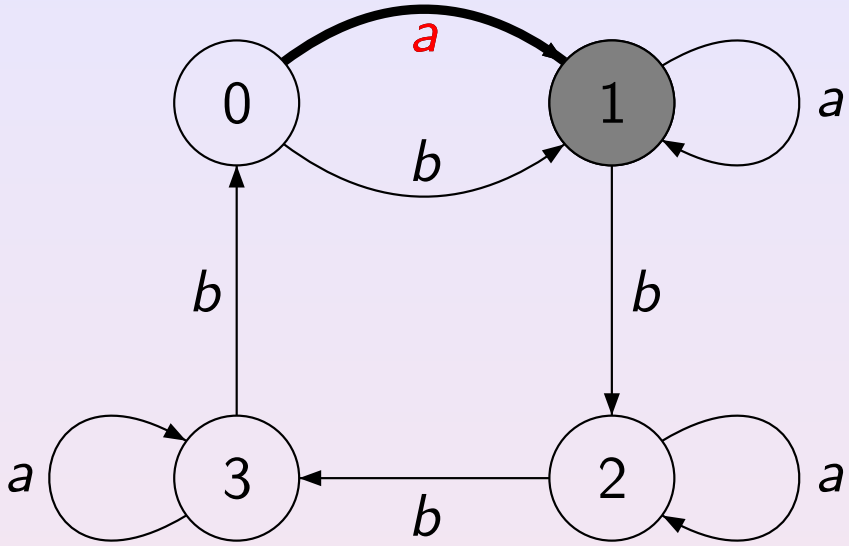

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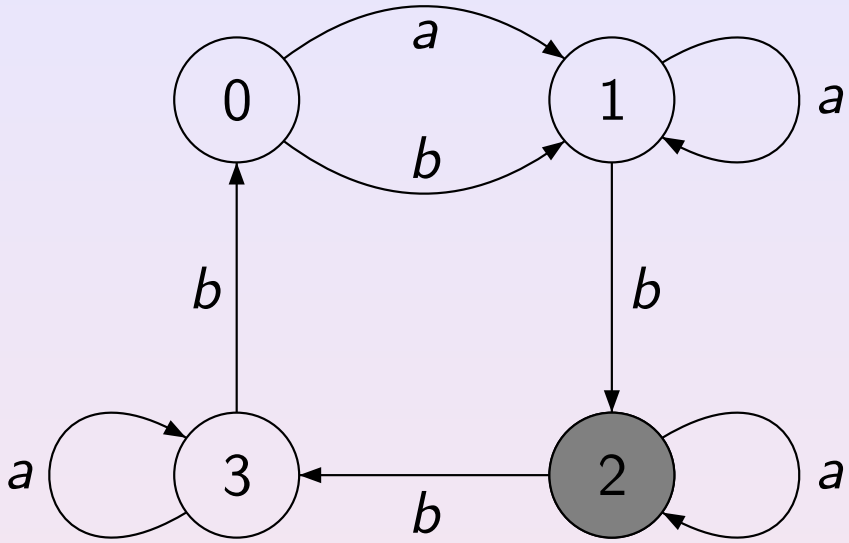

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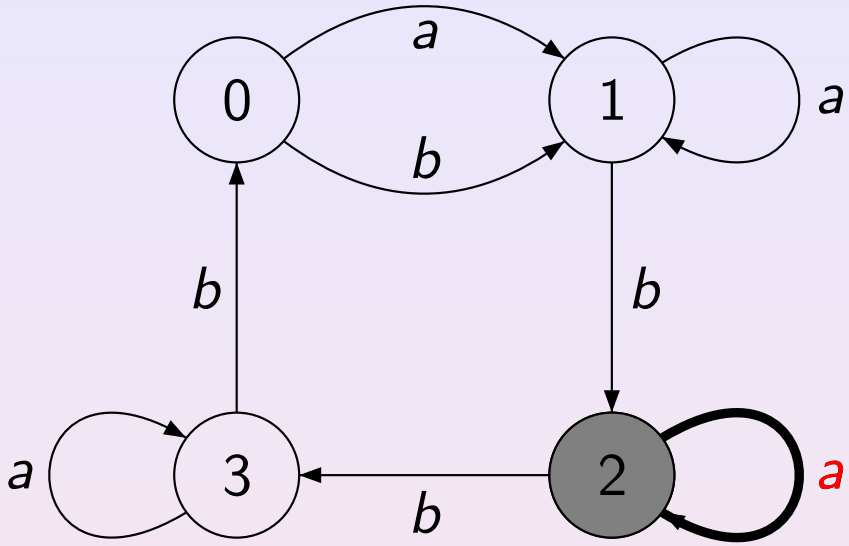

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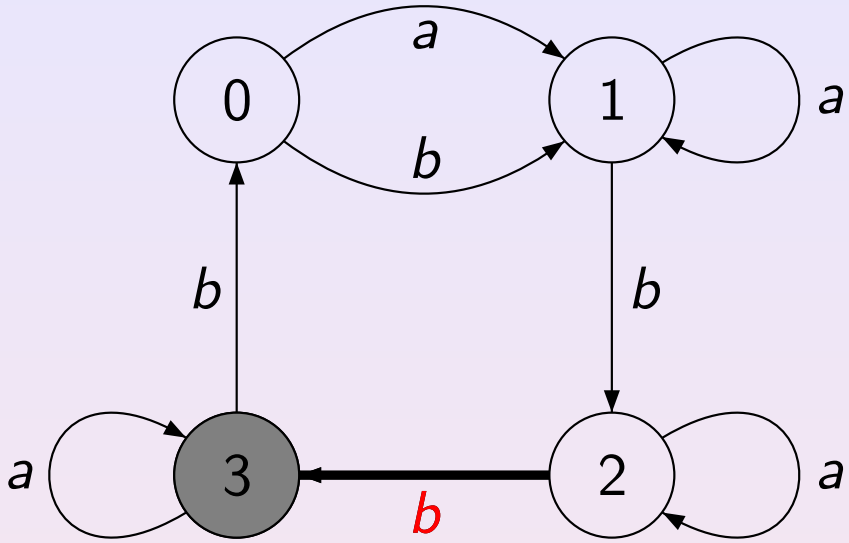

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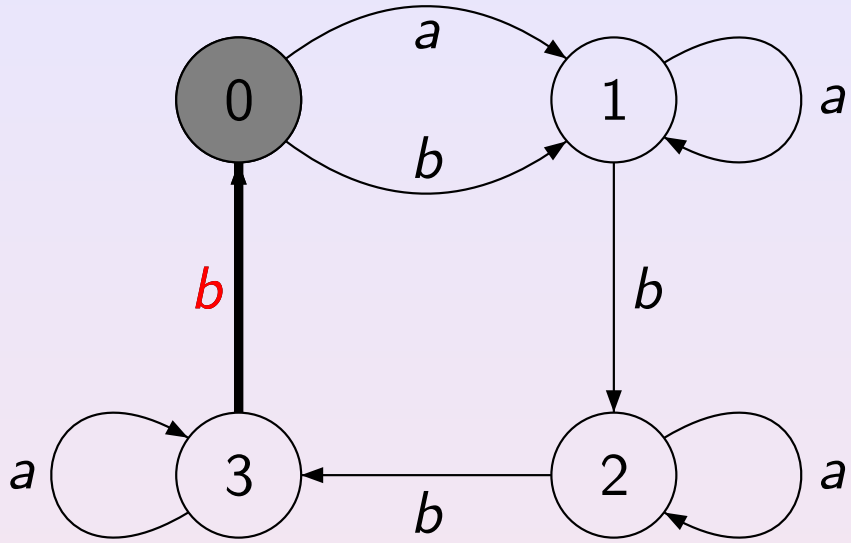

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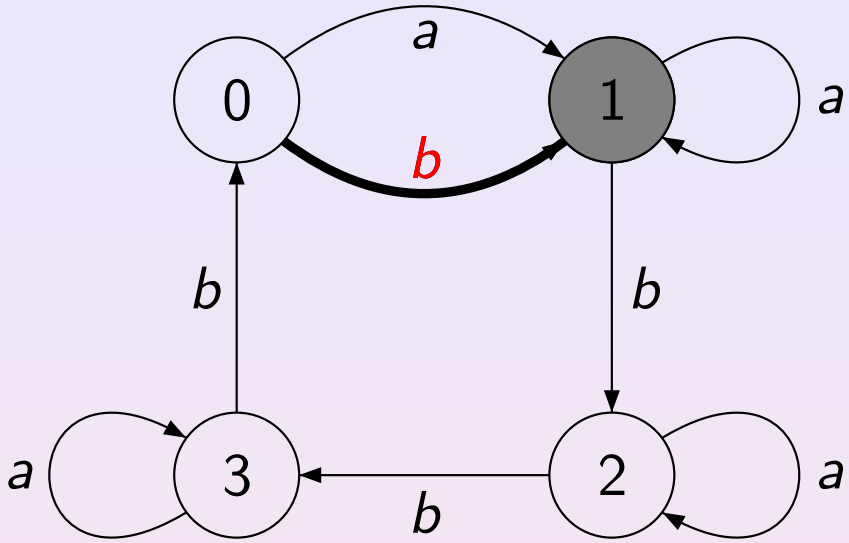

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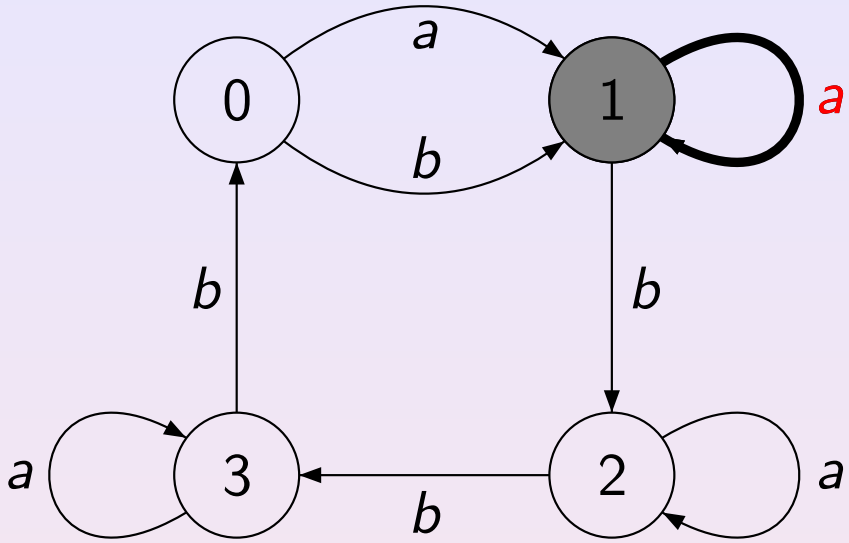

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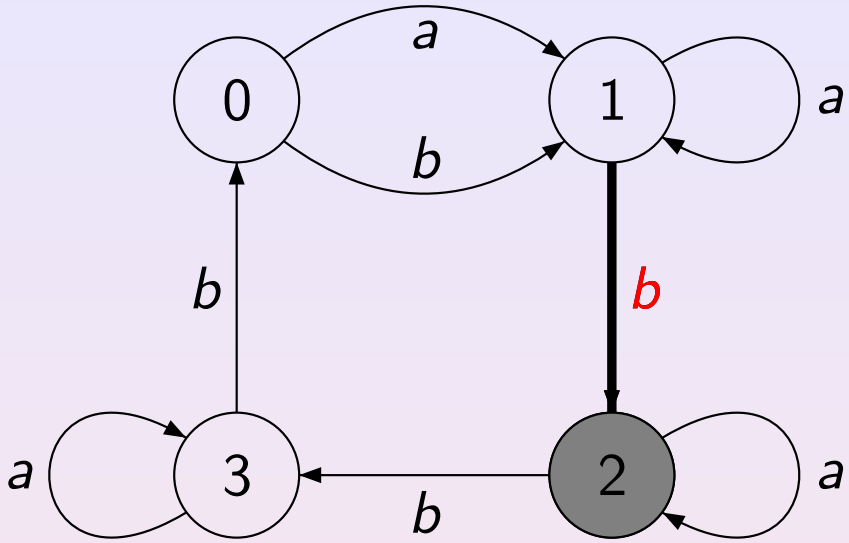

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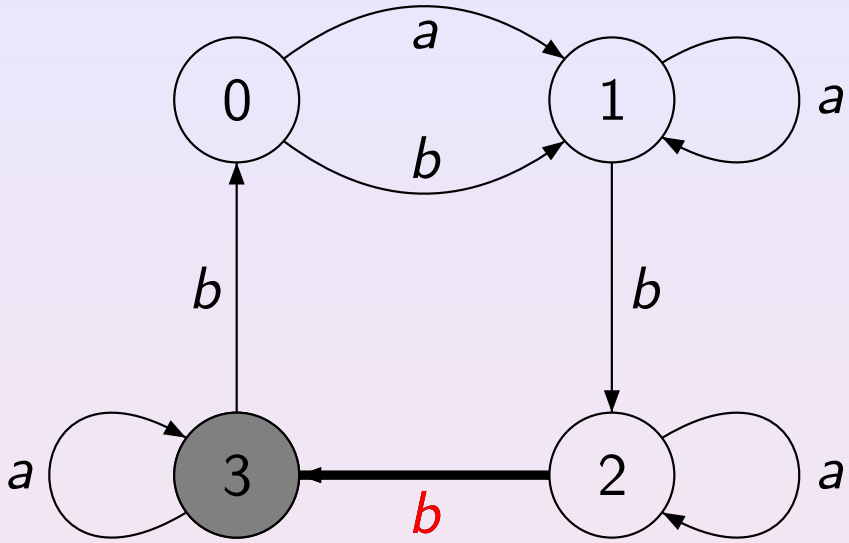

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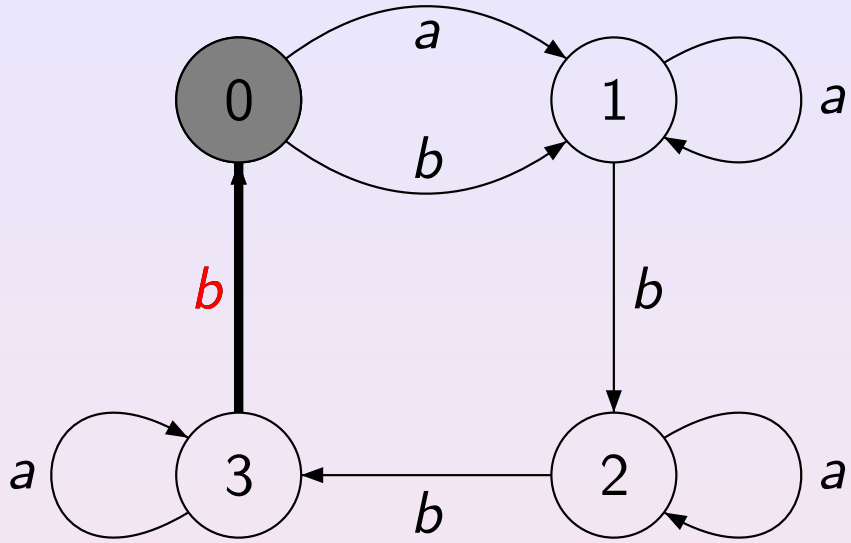

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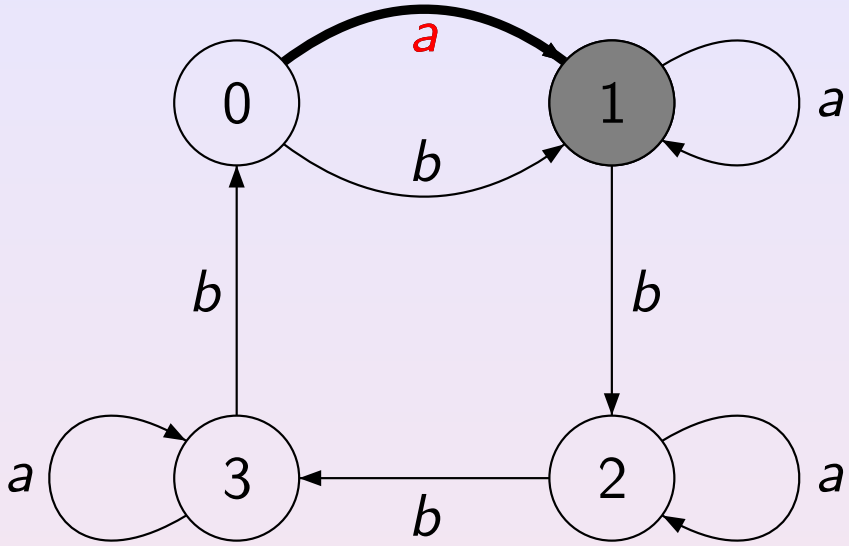

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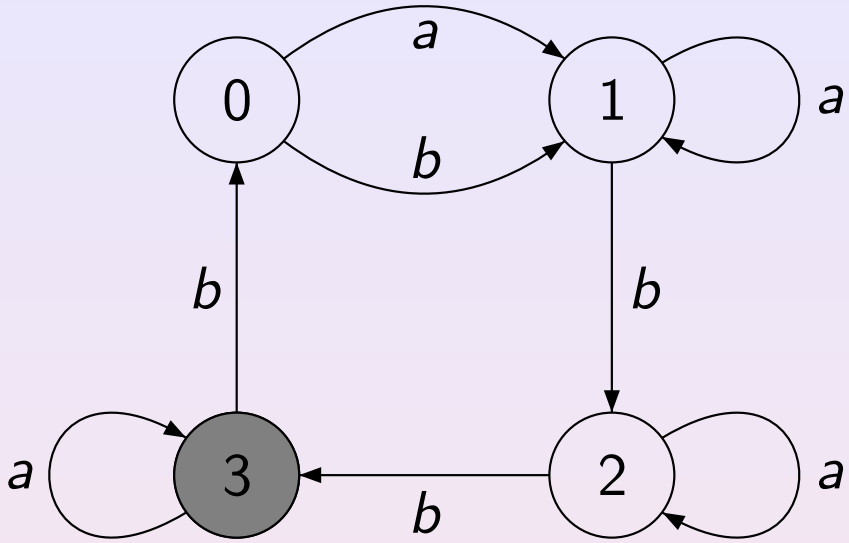

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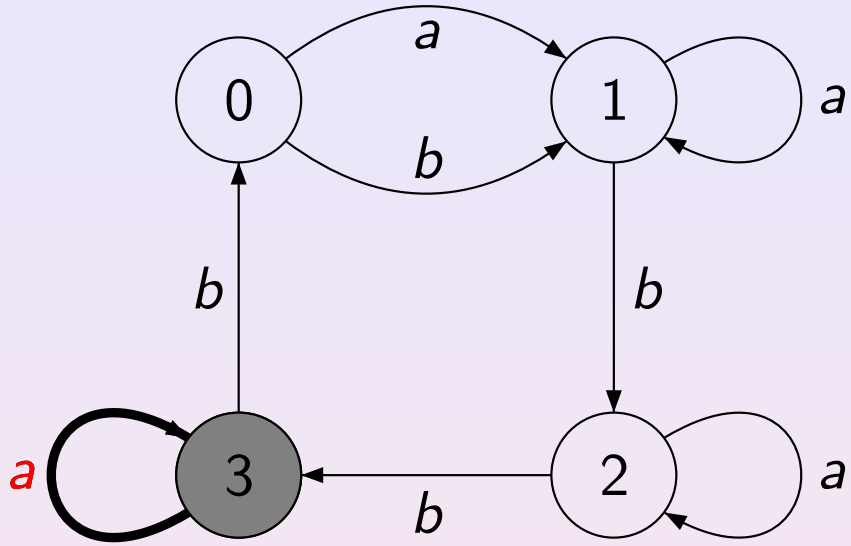

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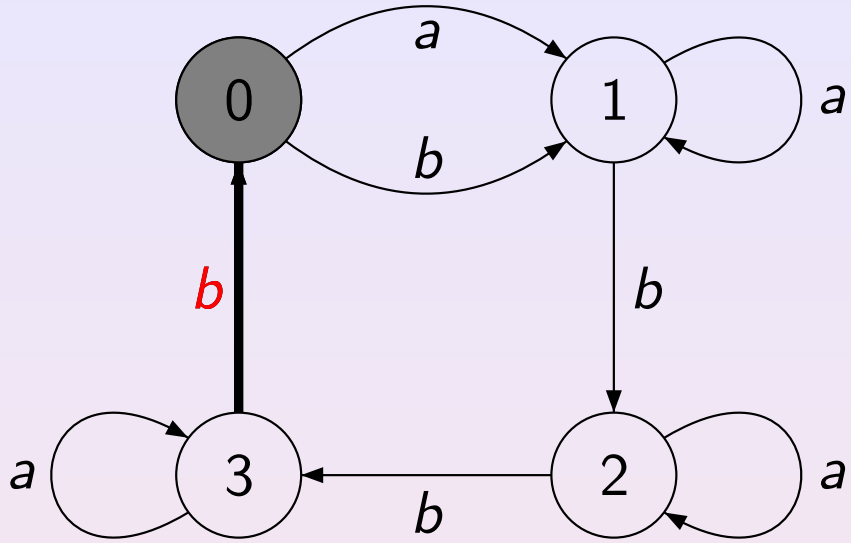

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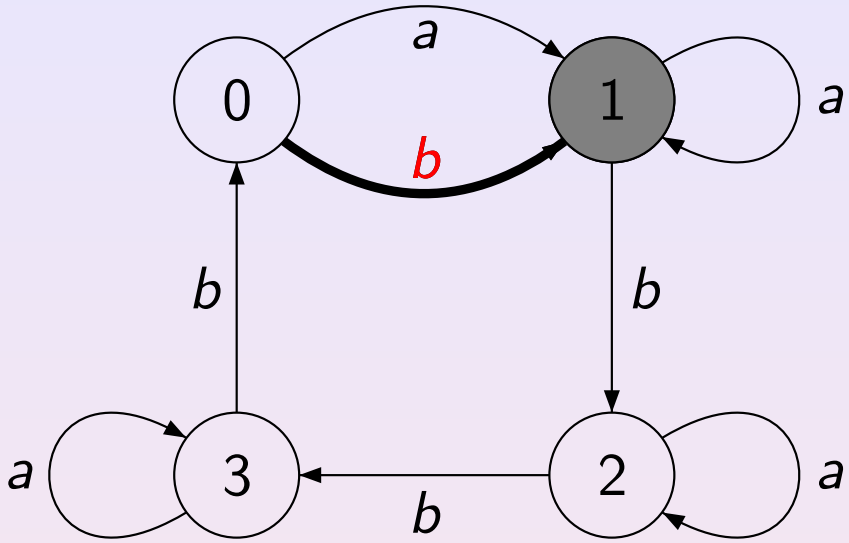

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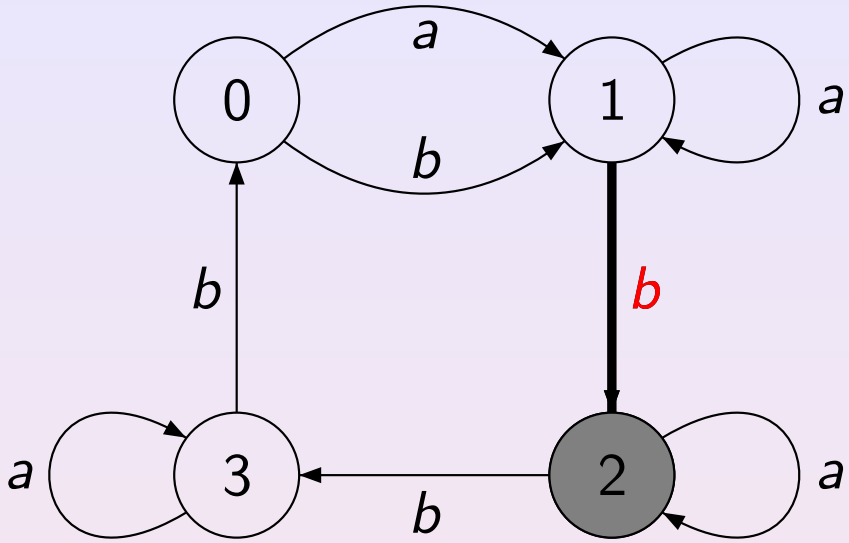

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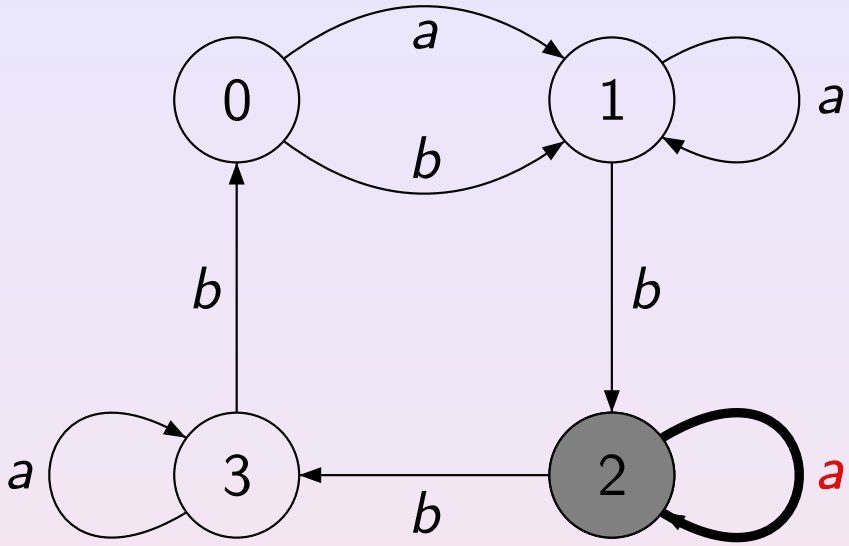

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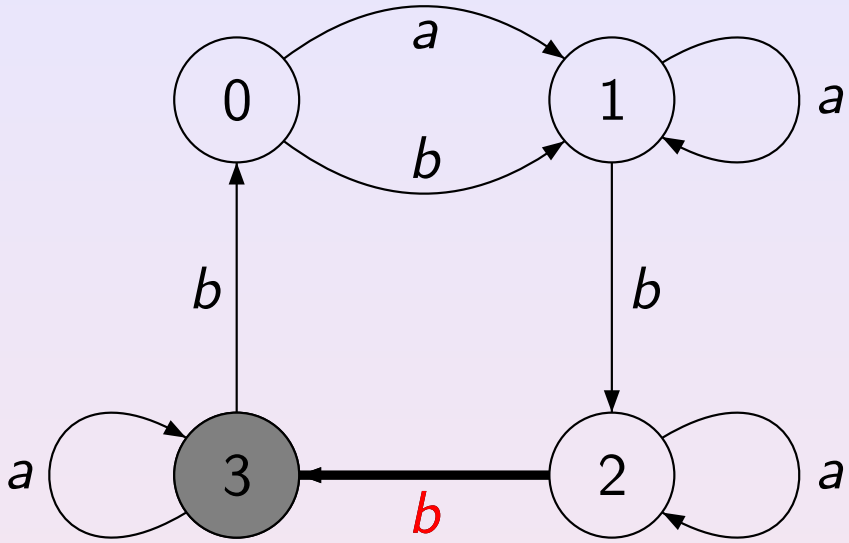

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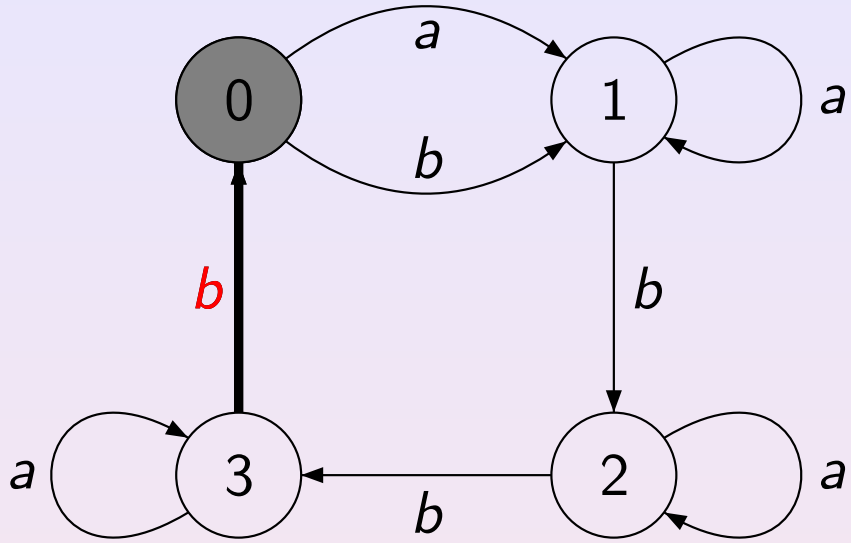

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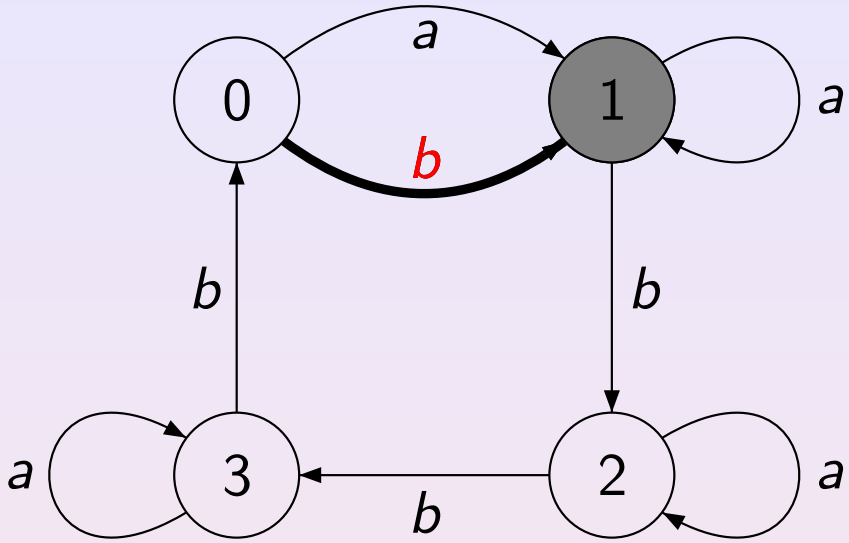

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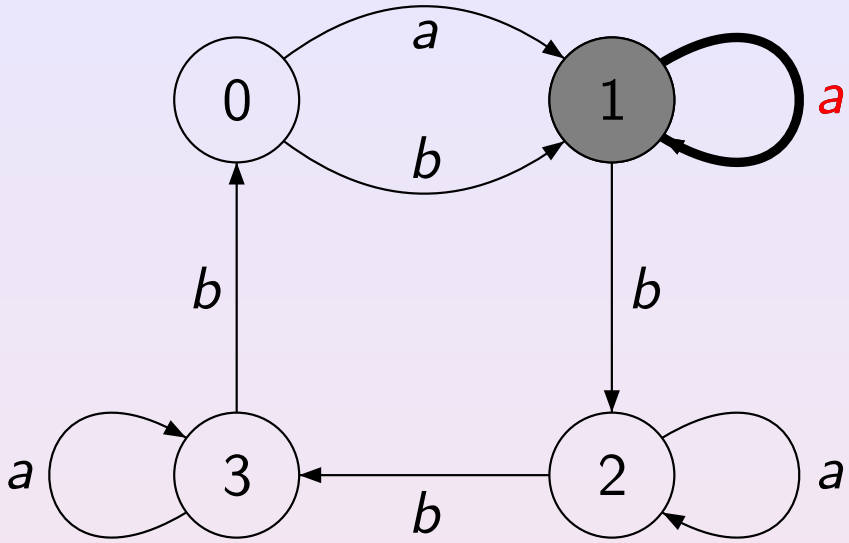

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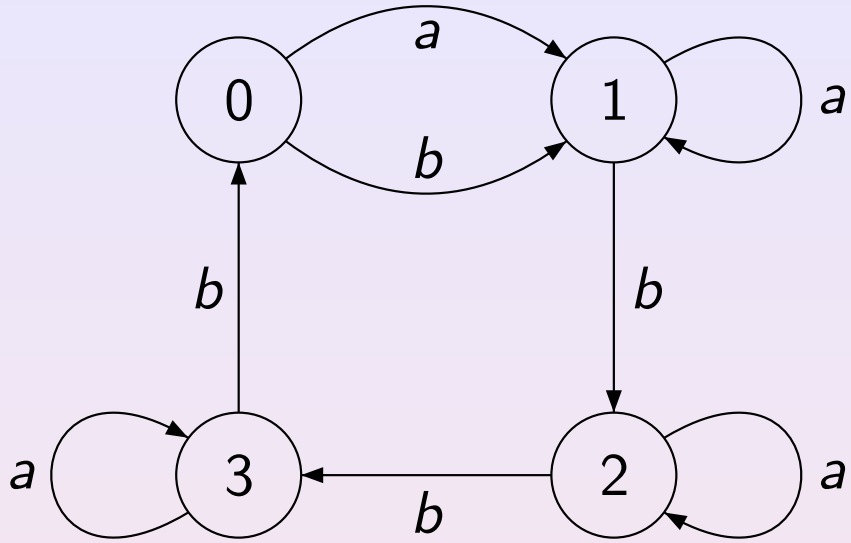

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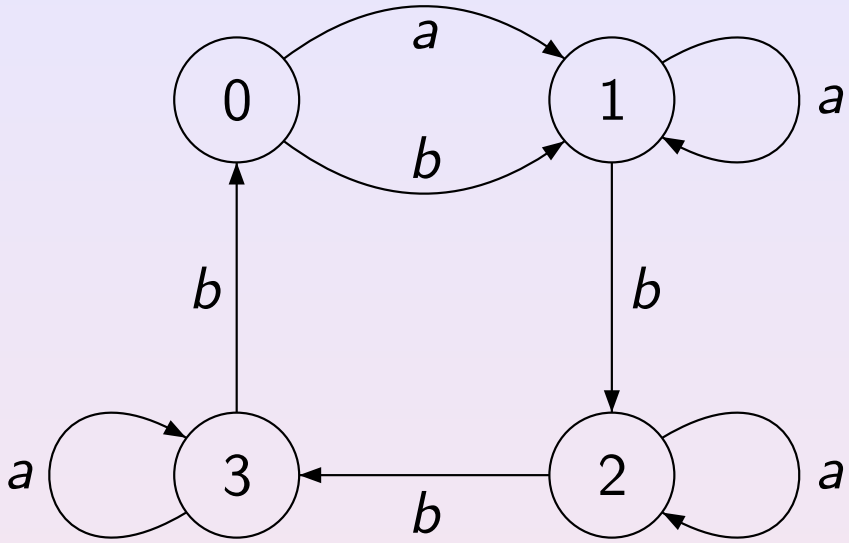

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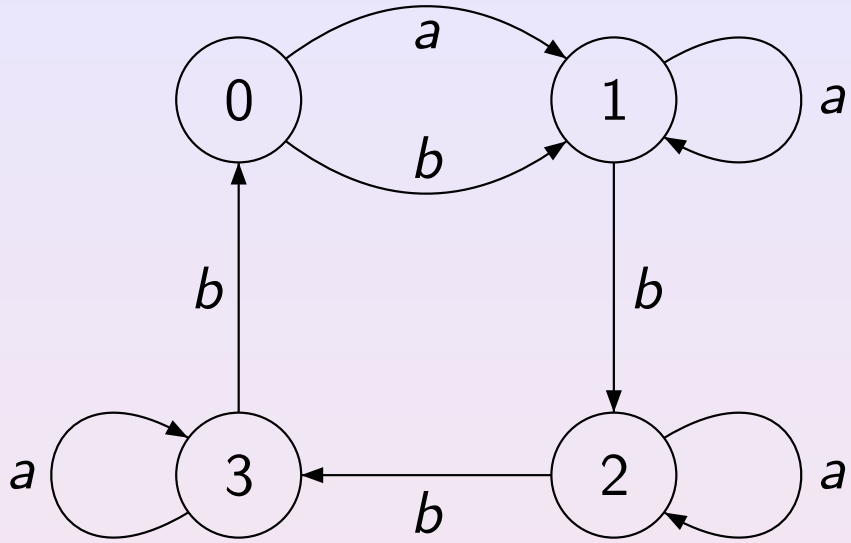

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The Černý Series

The states of $\mathscr{C}_n$ are the residues modulo n, and the input letters a and b act as follows:

$$0 \,.\, a = 1, \quad m \,.\, a = m \text{ for } 0 < m < n, \quad m \,.\, b = m + 1 \pmod{n}.$$

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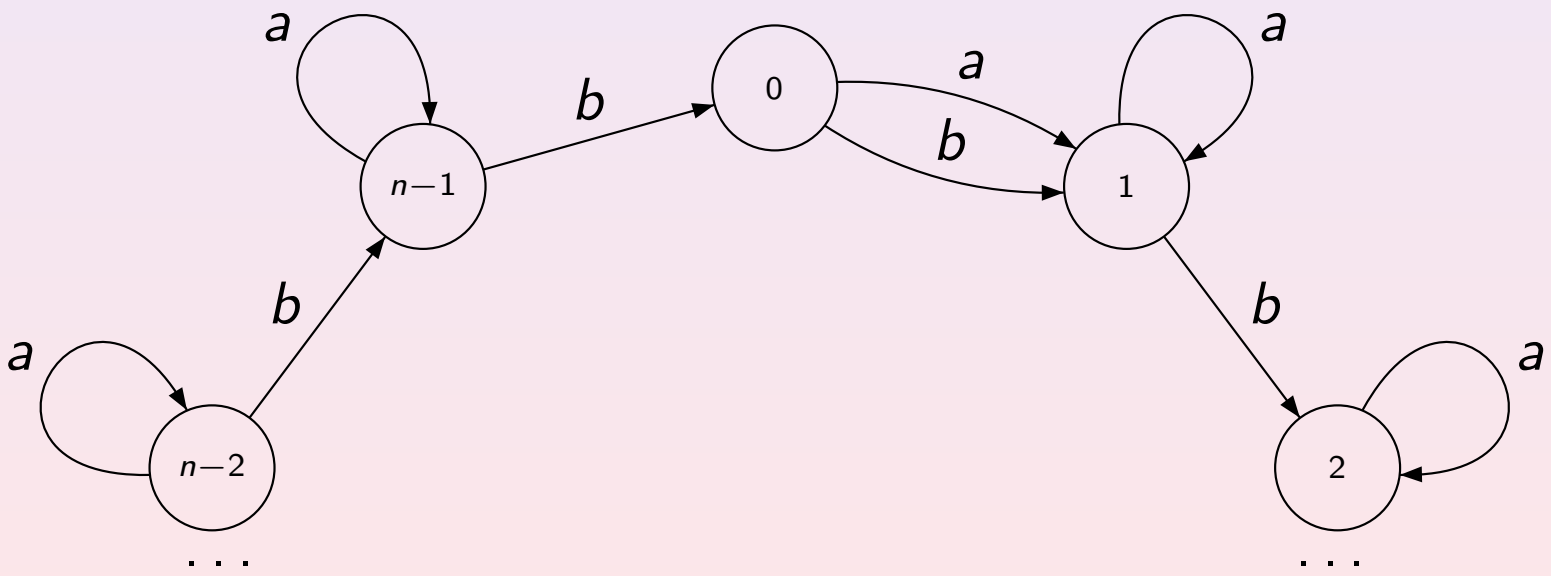
Here is a generic automaton from the Černý series:



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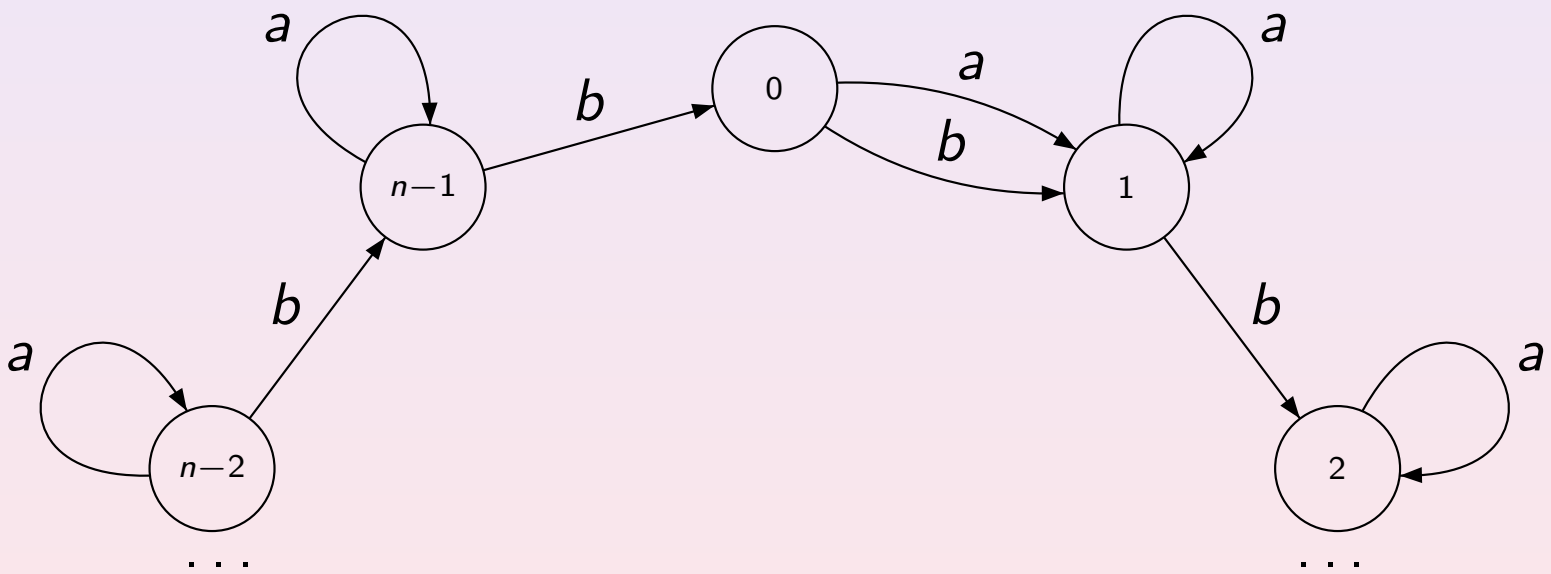

Černý has proved that the shortest reset word for $\mathscr{C}_n$ is $(ab^{n-1})^{n-2}a$ of length $n(n-2)+1 = (n-1)^2$.

The Černý Conjecture

The Černý conjecture is the claim that every synchronizing automaton with n states possesses a reset word of length $(n-1)^2$.

The validity of the conjecture is main open problem of the area and arguably one of the most long-standing open problems in combinatorial theory of finite automata.

Define the Černý function $C(n)$ as the maximum reset threshold for synchronizing automata with n states. In terms of this function, our current knowledge can be summarized in one line:

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The Černý conjecture thus claims that in fact $C(n) = (n-1)^2$.

Approaching the Černý Conjecture

Since the Černý Conjecture has proved to be hard in general, a natural strategy consists in considering its restriction to some special classes of DFAs.

The conjecture has been proved for many important special cases. This includes for instance:

- Louis Dubuc's result for automata in which a letter acts on the state set Q as a cyclic permutation of order $|Q|$ (Sur le automates circulaires et la conjecture de Černý, RAIRO Inform. Theor. Appl., 32 (1998) 21–34 [in French]).
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Approaching the Černý Conjecture

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For an illustration, consider the power-set automaton of the Černý automaton $\mathscr{C}_4$.

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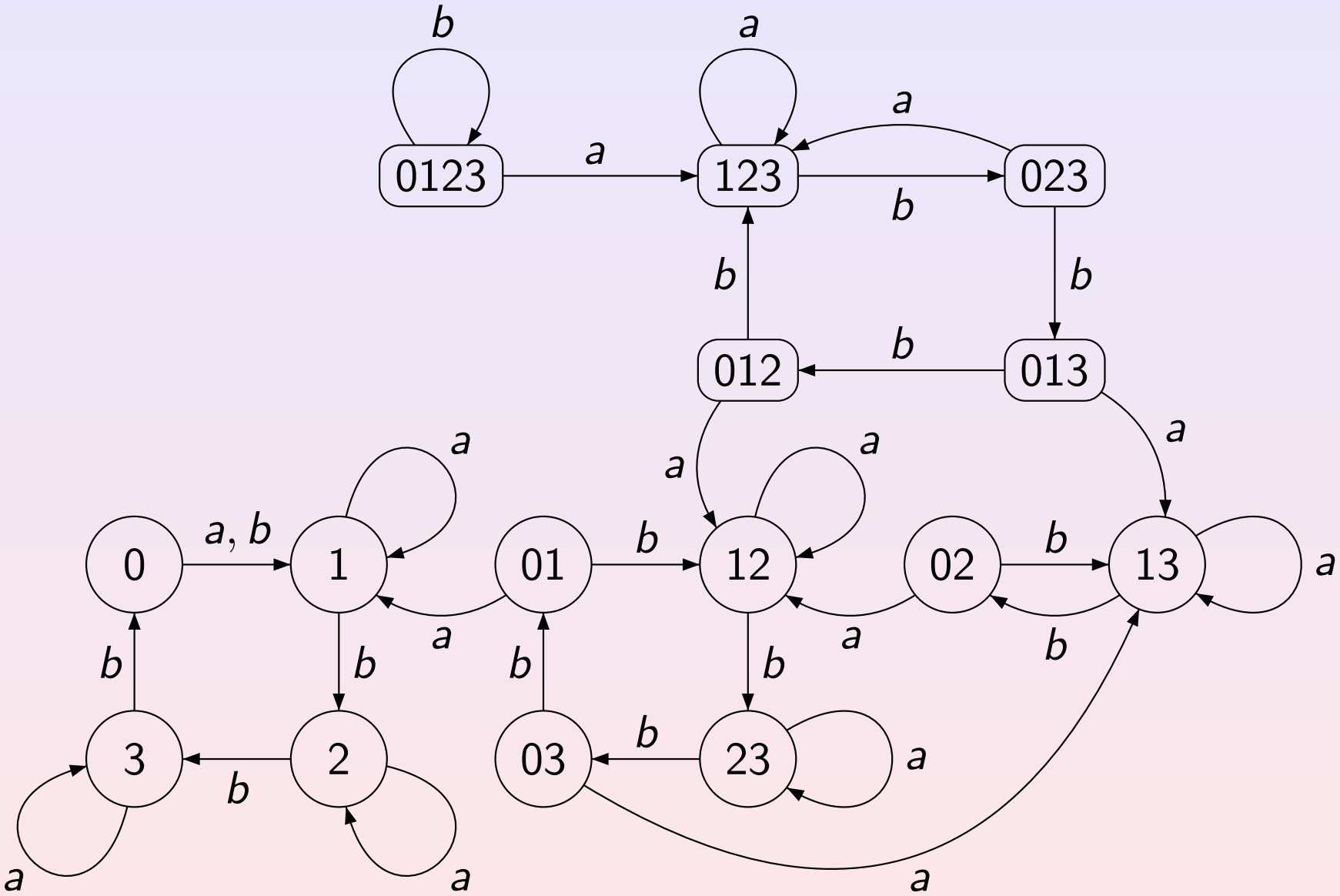

Restricting to Completely Reachable Automata

Recall that every completely reachable automaton is synchronizing.

On the other hand, the above observation ensures that the lower bound $(n-1)^2$ for the Černý function $C(n)$ is attained by a family of completely reachable automata.

Therefore completely reachable automata forms quite a natural class to study from the viewpoint of the Černý conjecture.

It is relatively easy to show that the conjecture holds true for completely reachable automata with two input letters; however, the general case remains open.

Some partial results in this direction have been obtained by Don.

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Second Motivation: Syntactic Complexity

Syntactic complexity of a regular language is a well established concept that has attracted much attention lately, in particular, in a nice series of recent papers by Janusz Brzozowski and his collaborators. It is defined as the size of the transition monoid of the minimal DFA recognizing the language. The transition monoid $M(\mathscr{A})$ of a given DFA $\mathscr{A} = \langle Q, \Sigma \rangle$ is the monoid of all transformations of the set Q induced by the words in Σ^*.

It appears to be worthwhile to extend this concept to automata by defining the syntactic complexity of an arbitrary DFA $\mathscr{A}$ as the size of its transition monoid $M(\mathscr{A})$. If one thinks of a DFA as a computational device rather than acceptor, its transition monoid can be thought of as the device's 'software library', and measuring the complexity of an automaton by the size of its 'software library' is fairly natural.

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Green's Relations

The following four equivalence relations can be defined on every monoid M:

$x \mathcal{R}\, y \Leftrightarrow xM = yM$, i.e., x and y are prefixes of each other;

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They were introduced by James Alexander Green in 1951 and since then they have played a crucial role in monoid theory and its applications to formal languages.

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One may think that the $\mathcal{L}$-trivial languages are left-right symmetric to the $\mathcal{R}$-trivial ones, but this is not the case! While the definitions of $\mathcal{L}$ and $\mathcal{R}$ are symmetric, the classes of $\mathcal{L}$-trivial and $\mathcal{L}$-trivial languages are not; in fact, the two classes are very different! Thus, the problem of determining the syntactic complexity of $\mathcal{L}$-trivial languages remains open.

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It is well known (and easy) that if two transformations from the transition monoid of a DFA are $\mathcal{L}$-related, they must have the same image. Hence, if a DFA $\mathscr{A}$ is such that different transformations from the monoid $M(\mathscr{A})$ have different images, then the monoid must be $\mathcal{L}$-trivial. How big can be the monoid with the latter property? Can the transition monoid of a completely reachable automaton have this property?

These questions are far from being obvious and are of interest also from the viewpoint of algebra, where the latter question has been stated in the literature as the question about the existence of $\mathcal{L}$-cross-sections in the full transformation monoid, see O. Ganyushkin, V. Mazorchuk, Classical Finite Transformation Semigroups: An Introduction. Springer, 2009.

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These questions are far from being obvious and are of interest also from the viewpoint of algebra, where the latter question has been stated in the literature as the question about the existence of $\mathcal{L}$-cross-sections in the full transformation monoid, see O. Ganyushkin, V. Mazorchuk, Classical Finite Transformation Semigroups: An Introduction. Springer, 2009.

Minimal Completely Reachable Automata

For every $n \geq 1$, we construct all completely reachable automata $\mathscr{A}$ with n states such that different transformations from $M(\mathscr{A})$ have different images. Clearly, the size of $M(\mathscr{A})$ is then $2^n - 1$, and this is the minimum possible syntactic complexity of a completely reachable automaton with n states. Therefore, we refer to these automata as to minimal completely reachable automata.

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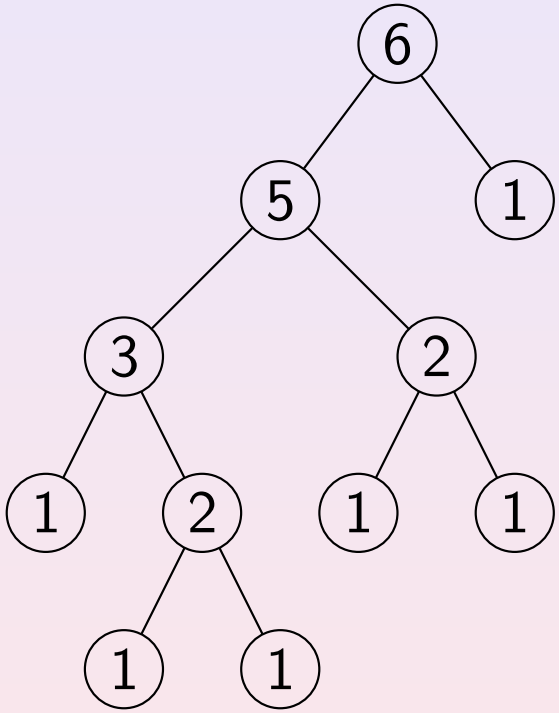
The span of v is the number of leaves in Γ_v.

Trees

This is a tree with vertices labelled by their spans:



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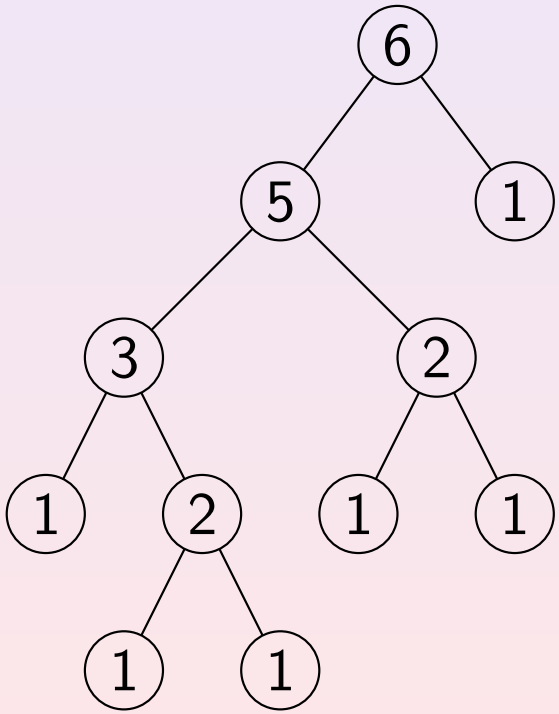
This tree is not respectful as it fails to satisfy (S2):



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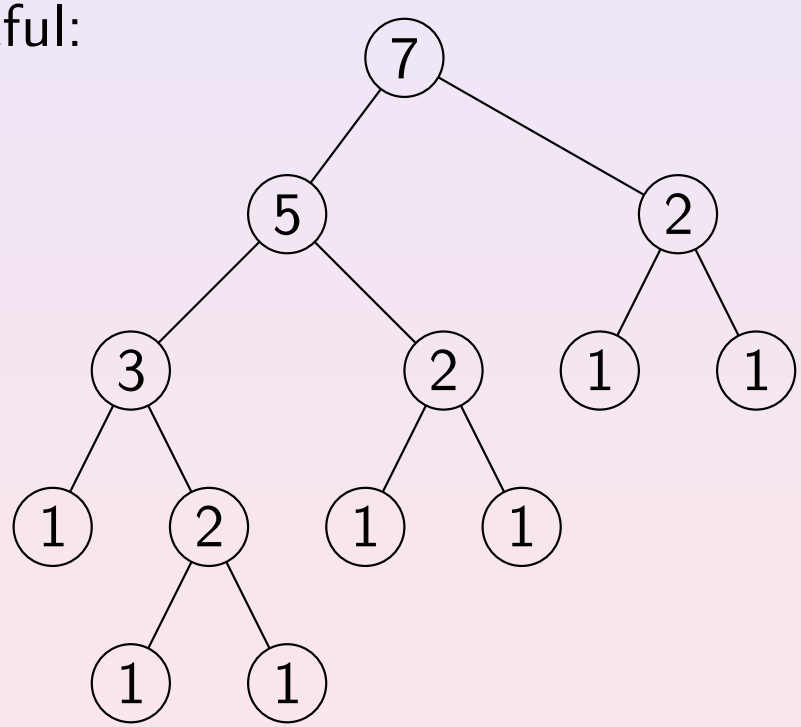

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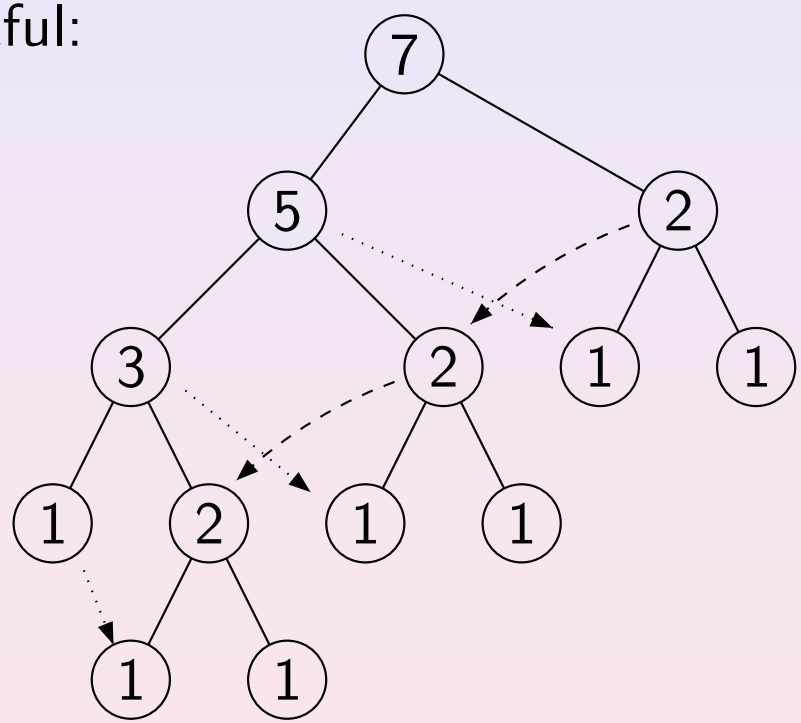

Here dotted and dashed arrows show the uncle–nephew and the aunt–niece relations respectively.

Interval Markings

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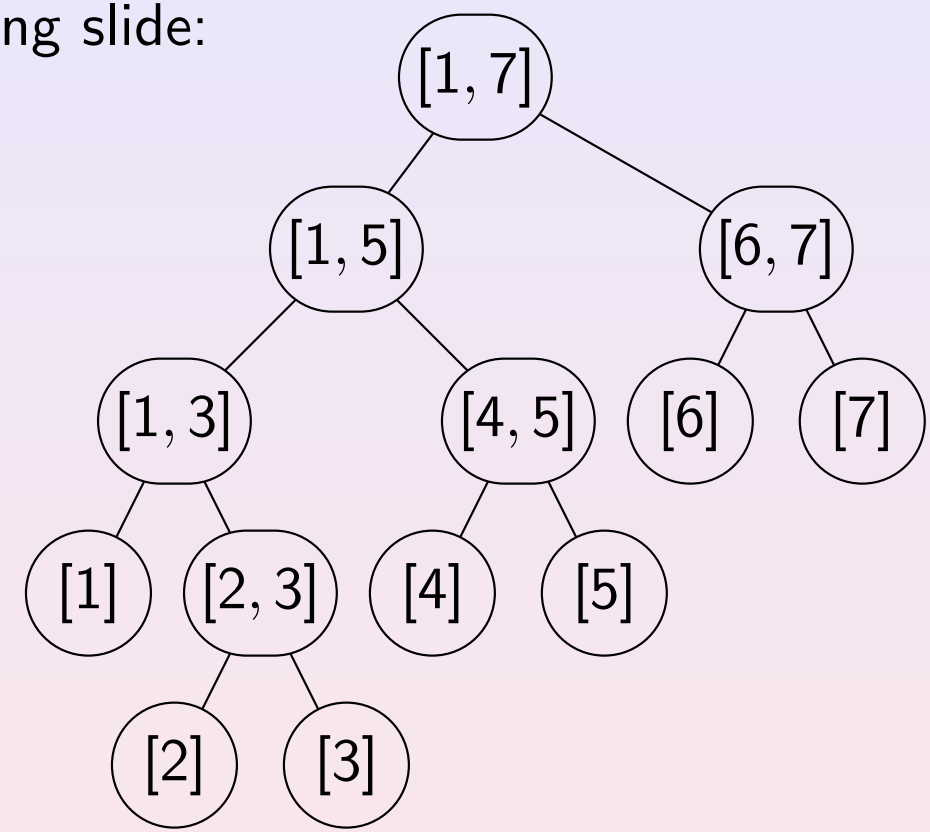
If μ is a faithful interval marking of Γ and v is a vertex of Γ, then the restriction of μ to the subtree Γ_v is a faithful interval marking of the latter.

Interval Markings

Here is a faithful interval marking of the respectful tree from the preceding slide:



Here we write $[i]$ instead of $[i, i]$.

From Trees to Automata

For each respectful tree Γ with n leaves and each its faithful interval marking μ, we construct an automaton $\mathscr{A}_\mu(\Gamma)$.

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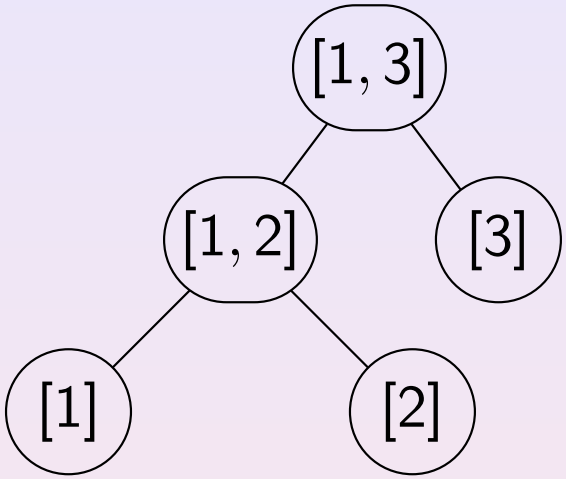
This completes our construction.

An Example



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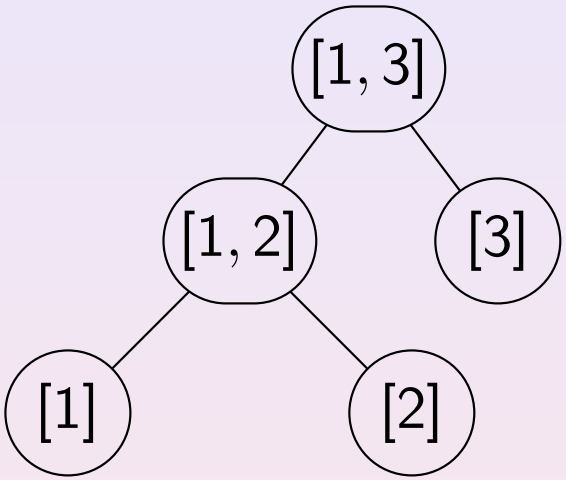

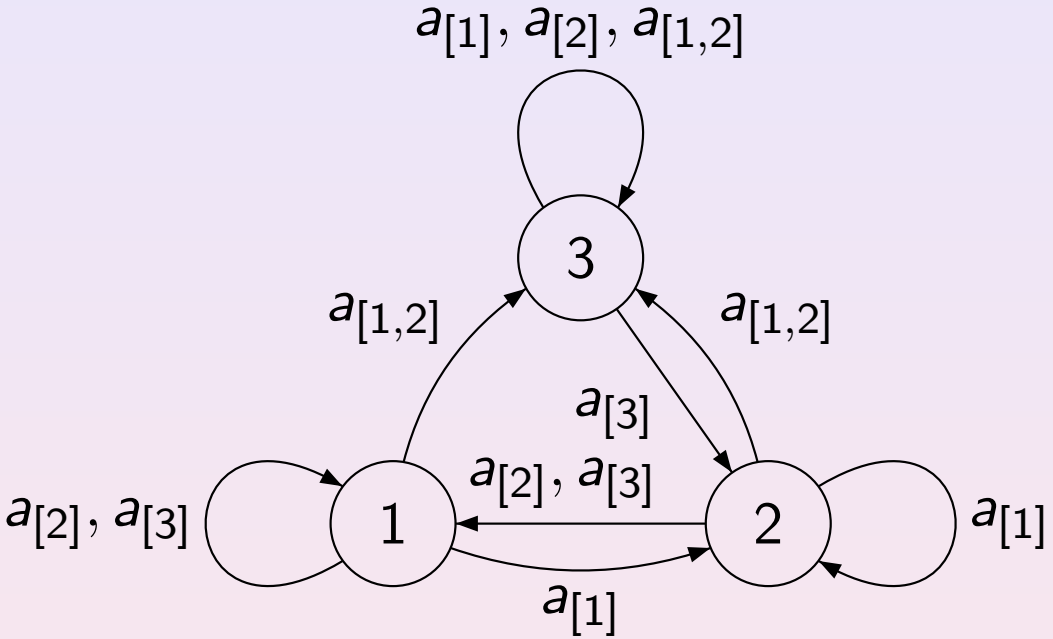

All Minimal Completely Reachable Automata

Automata constructed from different markings of the same respectful tree are isomorphic. Therefore we denote the DFA derived from any marking of a respectful tree Γ simply by $\mathscr{A}(\Gamma)$. We say that two DFAs $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ and $\mathscr{B} = \langle Q, \Delta, \zeta \rangle$ are syntactically equivalent if their transition monoids coincide.

1. For each respectful tree Γ, the automaton $\mathscr{A}(\Gamma)$ is a minimal completely reachable automaton.
2. Non-isomorphic respectful trees give rise to non-isomorphic automata.
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1. For each respectful tree Γ, the automaton $\mathscr{A}(\Gamma)$ is a minimal completely reachable automaton.
2. Non-isomorphic respectful trees give rise to non-isomorphic automata.
3. Every minimal completely reachable automaton is syntactically equivalent to an automaton of the form $\mathscr{A}(\Gamma)$ for a suitable respectful tree Γ.
4. Every minimal completely reachable automaton with n states has at least $2n - 2$ input letters.

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Three Open Questions

What are lower bounds for the syntactic complexity of completely reachable automata with restricted alphabet?

A DFA $\mathscr{A} = \langle Q, \Sigma, \delta \rangle$ induces a DFA $\mathscr{B} = \langle Q, \Delta, \zeta \rangle$ on the same state set if the transition monoid of $\mathscr{A}$ contains that of $\mathscr{B}$. This means that for every letter $b \in \Delta$, there exists a word $w \in \Sigma^*$ such that $\zeta(q, b) = \delta(q, w)$ for every $q \in Q$.

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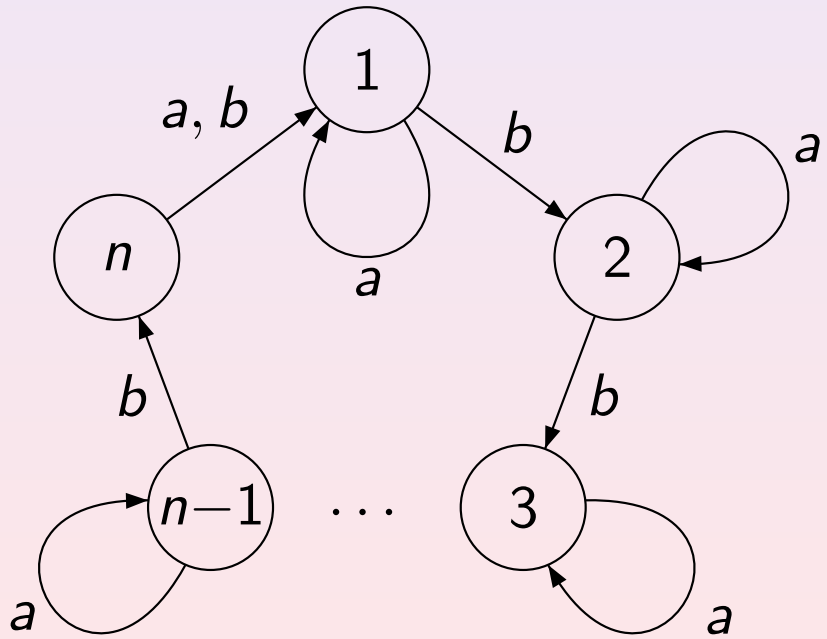

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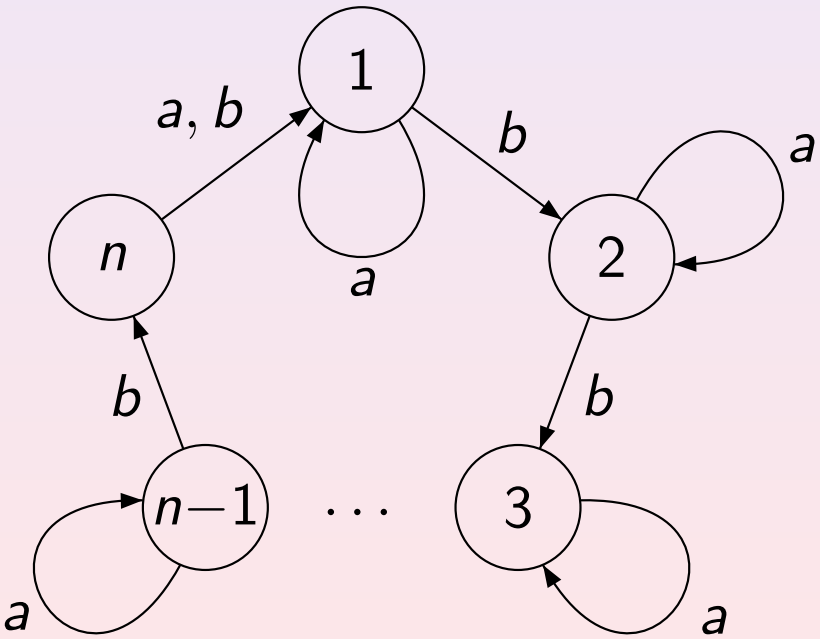

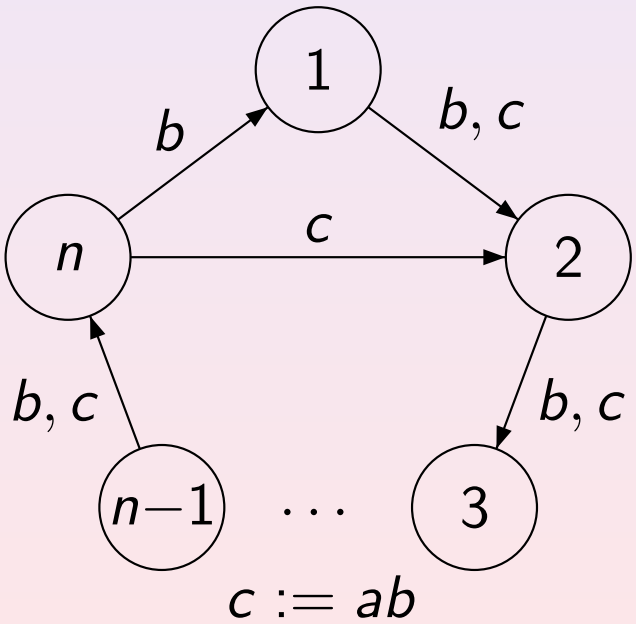

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What is the computational complexity of recognizing complete reachability?