The Finite Basis Problem for Kauffman Monoids

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The Finite Basis Problem (FBP) for a class $\mathscr{K}$ of algebras asks which algebras from $\mathscr{K}$ are finitely based and which are not.

Its algorithmic version for the class of finite algebras is known as Tarski's problem.

For general algebras (and even for groupoids) the corresponding problem is shown by Ralph McKenzie to be undecidable.

For semigroups it is still open and attracts lots of attention.

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FBP for Infinite Semigroups

The FBP for infinite semigroups is less studied.

Reason: “usual” infinite semigroups (transformations, relations, matrices etc) are too big (contain a copy of the non-monogenic free semigroup).

Therefore they satisfy only trivial identities and so they are finitely based in a “void” way.

But if an infinite semigroup does satisfy a non-trivial identity, its FBP constitutes a challenge since no “finite” methods apply.

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Neville Temperley and Elliott Lieb ("Relations between the percolation and colouring problem and other graph-theoretical problems associated with regular planar lattices: Some exact results for the percolation problem", *Proc. Roy. Soc. London* Ser. A 322, 251–280, 1971) motivated by some problems in statistical mechanics have introduced what is now called Temperley–Lieb algebras. These are associative linear algebras with 1 over a commutative ring R. Given n and $\delta \in R$, the algebra $TL_n(\delta)$ is generated by $n-1$ generators $h_1, \dots, h_{n-1}$ subject to the relations

$$\begin{aligned} h_i h_j &= h_j h_i \quad \text{if } |i-j| \geq 2, \\ h_i h_j h_i &= h_i \quad \text{if } |i-j| = 1, \\ h_i h_i &= \delta h_i. \end{aligned}$$

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Kauffman Monoids

The relations of $TL_n(\delta)$ do not involve addition. This suggests introducing a monoid whose monoid algebra over R could be identified with $TL_n(\delta)$. A tiny obstacle is the scalar δ in $h_i h_i = \delta h_i$. It can be bypassed by adding a new generator c that imitates δ. This way one gets to the monoid K_n with n generators $c, h_1, \dots, h_{n-1}$ subject to the relations

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The monoids K_n are called the Kauffman monoids. Lois Kauffman ("An invariant of regular isotopy", *Trans. Amer. Math. Soc.* 318, 417–471, 1990) independently invented these monoids as geometrical objects. We present Kauffman's approach in a slightly more general setting.

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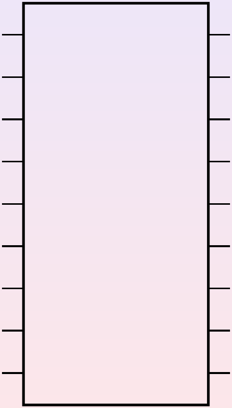
Wire Monoids

Fix n and consider "chips" with $2n$ pins, n on each side. Pins are connected by n wires. To multiply two chips, we connect the right hand side pins of the first with the left hand side pins of the second.



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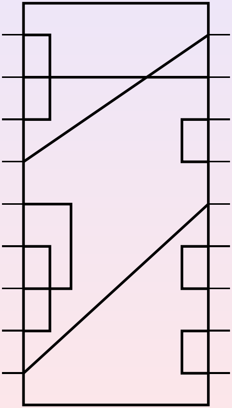

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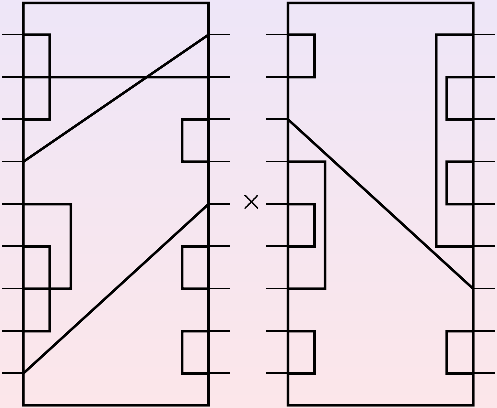

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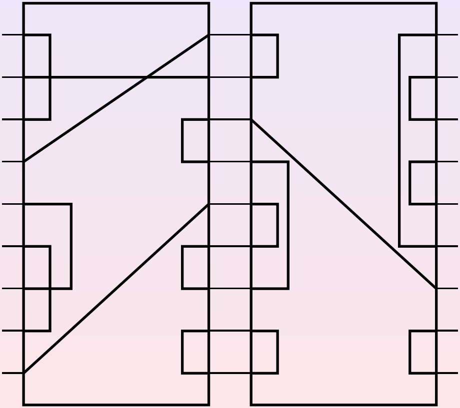

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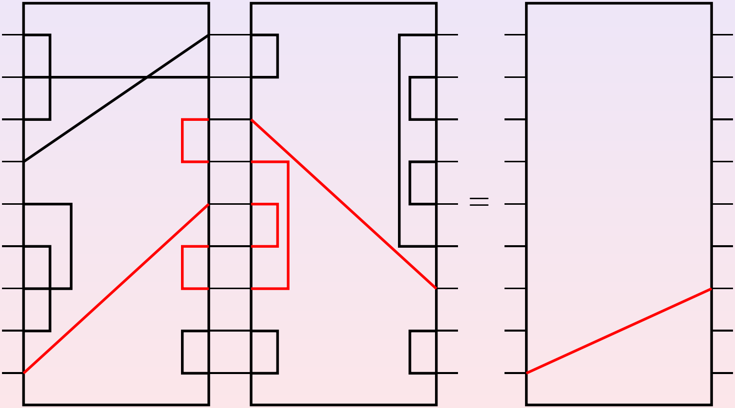

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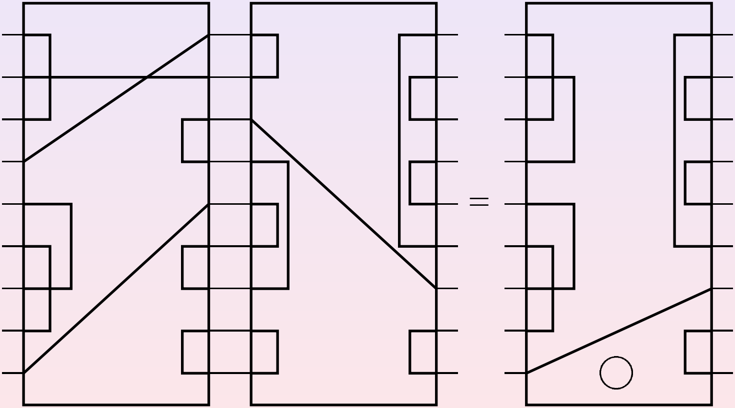

Types of Wire Monoids

There are two issues on which the outcome of the above definition depends: 1) do we care of crossing wires? 2) do we care of circles?

	Ignore circles	Count circles
Crossings OK	Brauer monoids	Wire monoids
No crossings	Jones monoids	Kauffman monoids

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Richard Brauer's monoids arose in his paper "On algebras which are connected with the semisimple continuous groups", *Ann. Math.* 38, 857–872, 1937 as vector space bases of certain associative algebras relevant in representation theory.

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Jones monoids are named after Vaughan Jones, the famous knot theorist. We denote by J_n the Jones monoid of chips with n pins.

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Kauffman Monoids as Wire Monoids

Thus the Kauffman monoid K_n consists of $2n$-pin chips with non-crossing wires that may contain circles. Only the number of circles matters, not their location. The monoid K_n is generated by the hooks $h_1, \dots, h_{n-1}$ and the circle c.

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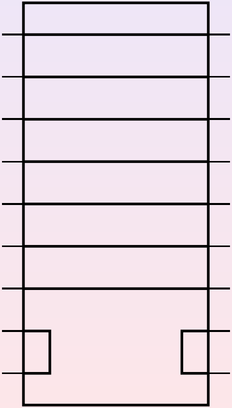

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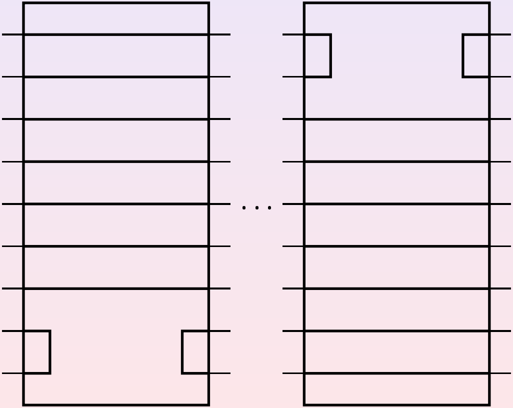

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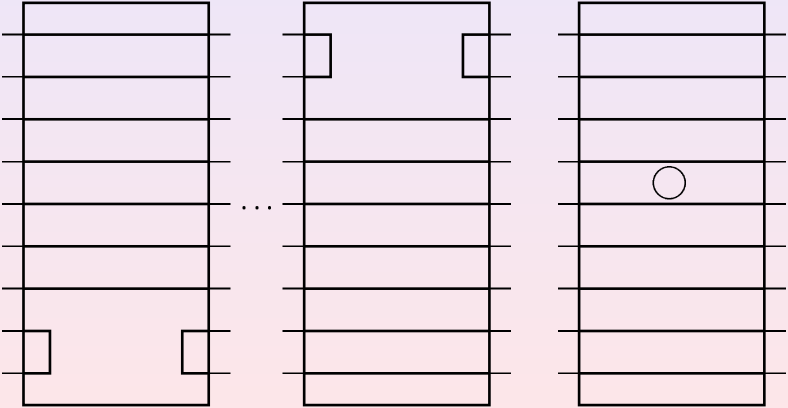

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These relations are satisfied when h_i and c are interpreted as the hooks and the circle. For the last relation it is clear—the circle does not react with the hooks, for the others it is shown in the next slides.

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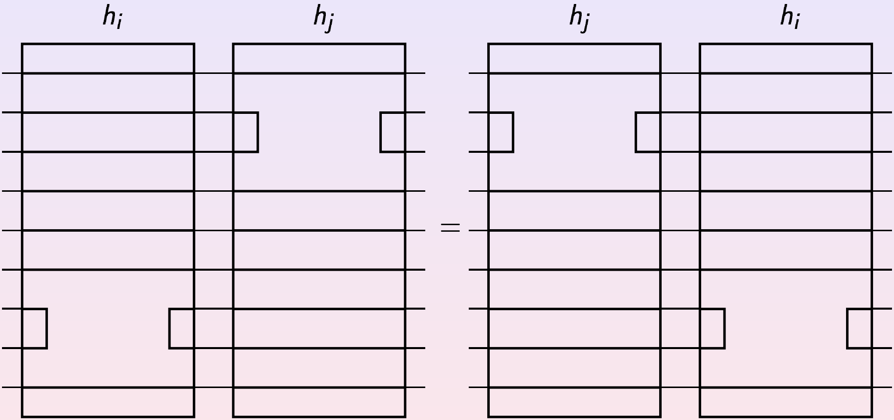

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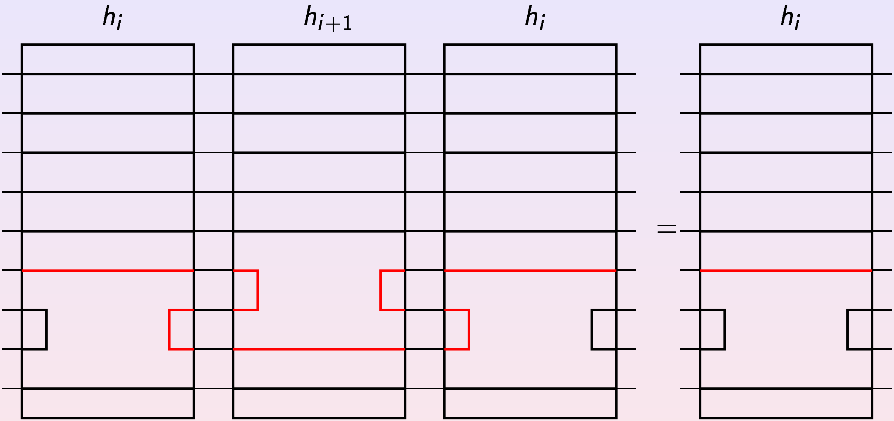

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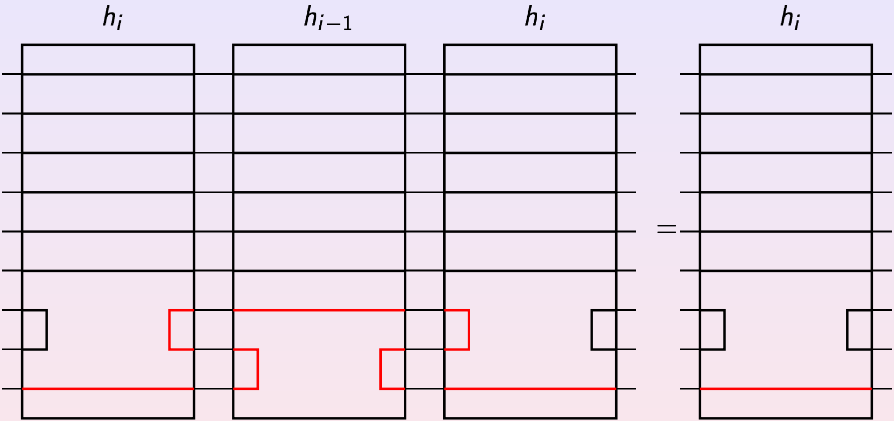
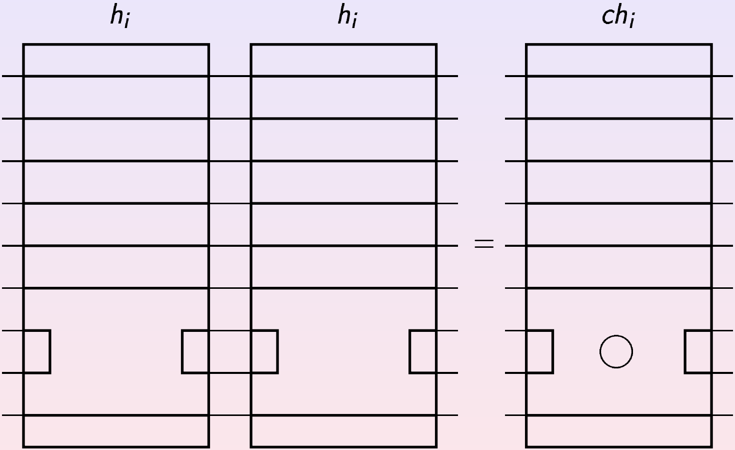


$h_i h_i = ch_i$



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Thus, the "planar" wire monoid generated by the hooks and the circle satisfies the relation of K_n and is therefore a homomorphic image of K_n. In fact, this wire monoid is isomorphic to K_n (requires some work). This connection was realized by Jones who didn't bother himself with a formal proof. Such a proof has first been published by Mirjana Borisavljević, Kosta Došen and Zoran Petrić ("Kauffman monoids", *J. Knot Theory Ramifications* 11, 127–143, 2002).

Similarly, one can show that the Jones monoid J_n is generated by the hooks $h_1, \dots, h_{n-1}$ subject the relations

$$h_i h_j = h_j h_i \quad \text{if } |i-j| \geq 2,$$
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Thus, J_n spans the Temperley-Lieb algebra $TL_n(1)$.

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Involutions

The full wire monoid admits two natural unary operations:
reflection (each chip reflects chip along its vertical symmetry axis) and rotation (each chip rotates by the angle of 180 degrees).
Both reflection and rotation are easily seen to be involutions, i.e.,

$$(xy)^* = y^* x^* \text{ and } x^{**} = x.$$

Clearly, the Kauffman monoid K_n is closed under each of the two involutions.

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FBP in Kauffman Monoids

Theorem

For each $n \geq 3$, the Kauffman monoid K_n is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

The Kauffman monoid K_2 is commutative and hence is finitely based; also as an involution semigroup. Hence we have a complete solution to the FBP for the Kauffman monoids in both plain and unary settings.

The fact that the monoid K_n with $n \geq 4$ is nonfinitely based was announced in my lecture at the 3rd Novi Sad Algebraic Conference in 2009. The case $n = 3$ was left open and so was the question about the FBP for K_n as an involution monoid with respect to reflection. Now these two questions have been settled + we have solved also the FBP for K_n as an involution monoid with respect to rotation.

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Further Applications

In fact, we have found a new sufficient condition under which a semigroup (or an involution semigroup) is nonfinitely based. When specialized to finite semigroups, it gives a well-known condition for a being inherently nonfinitely based. But there are many further applications to various classes of infinite semigroups.

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Theorem

For each $n \geq 3$, the full wire monoid (the monoid of all $2n$-pin chips) is nonfinitely based both as a semigroup and as an involution semigroup (with either of the two natural involutions).

Secret Slide

Let S be a semigroup [with involution] such that:

- S belongs to the Mal'cev product of the variety of all commutative semigroups [with the trivial involution] and a locally finite [involution] semigroup variety;
- each Zimin word Z_m is an [involution] isoterm relative to S.

Then S is nonfinitely based [as involution semigroup].

The Mal'cev product of varieties $\mathbf{V}$ and $\mathbf{W}$ is the class of all algebras A possessing a congruence θ such that A/θ lies in $\mathbf{W}$ while each θ-class being a subalgebra of A belongs to $\mathbf{V}$.

The Zimin words Z_m are defined as follows: $Z_1 = x_1, \dots, Z_m = Z_{m-1}x_mZ_{m-1}$.

A term t is an isoterm relative to an algebra A if the only term t' such that the identity $t = t'$ holds in A is t itself.

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